Material Provided by EPA in response to questions from the SAB regarding the limitations of bounding analyses in quantitative uncertainty analyses

Bounding versus Uncertainty Analysis

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This note responds to a request from the Science Advisory Board (SAB) for the dioxin reassessment to clarify why a bounding analysis is of limited usefulness in the context of quantitative uncertainty analysis. The general outlines of quantitative uncertainty analysis have not changed substantively since the codification in the U.S. Nuclear Regulatory Commission (NRC) Guide for probabilistic risk analysis (PRA) (U.S. NRC, 1983). Indeed, the breakdown of the chapter on Uncertainty and Sensitivity Analysis still serves as a good introductory roadmap (see appendix). With regard to bounding or "simple interval measures" the PRA Procedures Guide states:

> *The simplest quantitative measure of variability in a parameter or a measurable quantity is given by an assessed range of the values the parameter or quantity can take. This measure may be adequate for certain purposes (e.g., as input to a sensitivity analysis), but in general it is not a complete representation of the analyst's knowledge or state of confidence and generally will lead to an unrealistic range of results if such measures are propagated through an analysis. (p. 12-12)*

These same concepts are widely represented throughout the uncertainty analysis literature. According to Morgan and Henrion (1990):

> *Uncertainty analysis is the computation of the total uncertainty induced in the output by quantified uncertainty in the inputs and models…. Failure to engage in systematic sensitivity and uncertainty analysis leaves both analysts and users unable to judge the adequacy of the analysis and the conclusions reached. (p. 39)*

More recently, the National Research Council's scientific review of the Office of Management and Budget's proposed risk assessment bulletin (NRC, 2007) notes that the Bayesian interpretation for representing probability and Monte Carlo methods of propagation are among the methods in the evaluation of uncertainty in risk analysis "about which there is near unanimity in the scientific community" (p. 47).

It is not difficult to understand why this method leads to unrealistic results. Figure 1 shows the sum of 20 variables uniformly distributed on the [0, 1] interval. The maximal value is attained if all 20 variables take their maximal value, 1. Similarly, the minimal value is obtained if all variables take their minimal value of 0. A bounding analysis giving the smallest interval which is certain to contain the true value of the sum is [0, 20].

The probability that those extreme values would be realized is negligible. If the variables are independent, there is a 0.1% chance (1 in a 1000) that the value of the sum is less than 6.07 or greater than 13.9 (see Figure 1). Dependence can dramatically alter the picture. A modest positive dependence between the 20 variables results in the second set of 3 bands; the 90% band in this case [3.61, 16.39] is greater than the 98% band in the independent case. Negative dependence (half the variables correlated at -0.9 with the other half) leads to shrunken bands. From this simple example we conclude (1) upper and lower bounds give an unrealistic picture, and (2) the dependence structure between the variables is important.

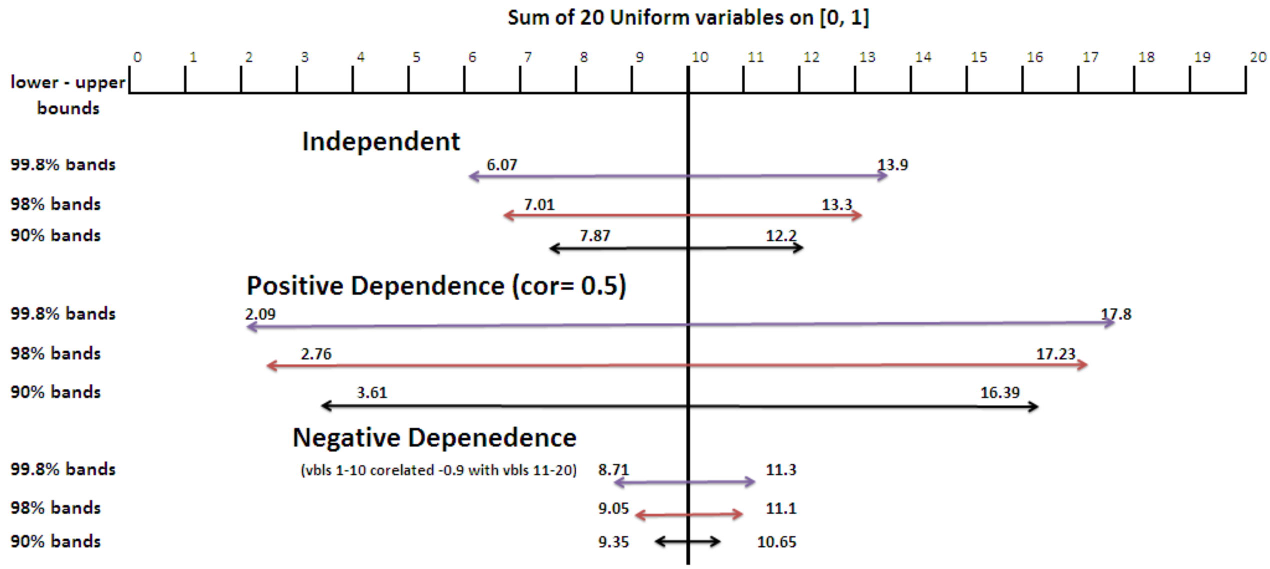

Figure 1 represents an extremely simple case in which the upper and lower bounds are readily apparent, and the number of variables is very small. If the variables were combined in more complex functions, it would not be possible to infer the upper and lower bounds by inspection. Instead these would be estimated by Monte Carlo simulation, i.e. by sampling the intervals a large number of times and estimating the bounds from the maximum and minimum sample values. Any continuous joint distribution supported on the input intervals will (eventually) yield the same maxima and minima for functions of the input, however, some distributions will be more efficient than others in this regard. Since knowing which distributions are most efficient is just as hard as knowing which combinations of input values maximize or minimize a function, the only practical option is simply assigning independent uniform distributions to the input intervals and sampling profusely. From this perspective a bounding analysis is just a reduced form of uncertainty analysis in which the distributions placed on the input intervals are not material. The endpoints of the input intervals, however, are *very* material. The input intervals must contain the true values with absolute certainty.

In some cases there may be obvious bounding intervals for the input variables. For example, a percentage is necessarily confined to the interval [0, 100]. In other cases there may be physical limits on the variable. Thus, the lateral spread of a plume of airborne pollutants at a given time after release is between 0 and the circumference of the earth. Physical limits may be implausibly wide, yet specifying an interval which captures all pollutants with absolute certainty may also be difficult and may still lead to very wide bounds. In any event, measurement error bars or expert confidence ranges do not have the property that the true value falls within these bounds with absolute certainty.

Some authors would object to the use of the uniform distribution in the above context. The "p-box" interpretation holds that intervals must contain the true values with full confidence, but within each interval the uniform distribution is inappropriate, since: "The interval lacks any concentration of probability or likelihood within the interval, so the actual value is not more likely to be at any one place or another within the interval. But neither is there necessarily a uniform probability distribution over the interval. Instead the actual value has complete latitude to be anywhere within the interval with probability one" (Ferson et al. 2007, p. 19). On this view, "the actual value is not more likely to be at any one place or another within the interval" and "each possible value in an interval is equally likely" (the "equidistribution hypothesis") are incompatible "perspectives on incertitude" (Ferson et al. 2007, p. 33).

References

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Appendix *(excerpted from U.S. NRC, 1983)*