Preliminary Individual Comments

Computable General Equilibrium Model Review Panel

Contents

Comments from Dr. Alan Fox 2
Comments from Dr. Mun Ho 6
Comments from Dr. David Montgomery 12
Comments from Dr. Sergey Paltsev 18
Comments from Dr. Thomas Rutherford 21
Comments from Dr. Ron Sands 23
Comments from Dr. Dominique van der Mensbrugghe 26
Comments from Dr. Peter Wilcoxen 32
Appendix 1 ..... A1 – 1
Appendix 2 ..... A2 - 1

Comments from Dr. Alan Fox

1. *Is the model documentation clear, accurate, and transparent? Do you have any specific suggestions for how to improve it?*

    a. Regarding installation and building the benchmark database: I had a number of difficulties building the benchmark database due to the fact that my work computer is located behind a proxy. The problem was ultimately resolved by building the model on my home computer, without a proxy (though behind a firewall). In consultation with Alex Marten, we partially resolved some of my proxy-related problems, though not all of them. Here are a few notes related to the solution.

    b. Important notes regarding software installation (perhaps obvious, but worth including in documentation):

        i. Ensure that GAMS is on the search path. The R script cannot find the GAMS executable otherwise, and it does not appear to throw an error when it fails.

        ii. If you are behind a proxy, be sure to include appropriate proxy lines in your .Renviron file and .Rprofile file. Text in <> should be replaced with appropriate values below:

```
# Proxy settings in .Renviron file

http_proxy=http://<ProxyAddress>:<ProxyPort>/
http_proxy_user=<UserName>:<UserPassword>

https_proxy=https://<ProxyAdress>:<ProxyPort>/
https_proxy_user=<UserName>:<UserPassword>

# Proxy settings in .Rprofile file:

library(httr)
httr::set_config(config(proxy="<ProxyAddress:<ProxyPort>",
proxyuserpwd = "<UserName>:<UserPassword"))
```

c. I have a number of minor editorial comments (which I can forward if useful), but one overarching suggestion concerns how the LaTeX math equations are typeset. In cases where multi-character variable names are employed, especially those including the letter *f* (e.g., in equation 23, *tfica*, *bopdef*, *refund*), you should use \mathit{} so they typeset properly. Fox example, pfx_{t}bopdef_{t,r,h} should be set as \mathit{pfx}_{t}\mathit{bopdef}_{t,r,h}. As it is, there is too much space preceding the "*f*". This is especially noticeable in the case of "*tfica*", where the "fi" has a ligature when set using \mathit{} and none otherwise. If you do this for all multi-letter variables, it reads easier, because it signals LaTeX to tighten up the spacing within the variable name, but appropriately puts spacing between variables.

BEFORE:

```
 \begin{equation}
  \begin{alignedat}{1}
   kh_{t+1,r,h}+pcl_{t,r,h}cl_{t,r,h} = &
    \left(1+r_{t}\right)kh_{t,r,h}+\left(1-tl_{t,r,h}-tfica_{t,r,h}\right)pl_{t,r}te_{t,r,h}\\
    & +pr\_ex\_agg_{t,r}kh\_ex_{t,r,h} +\sum_{s}{pres_{t,r,s}rese_{t,r,s,h}}\\
    & +pfx_{t}bopdef_{t,r,h}+cpi_{t}tran_{t,r,h}\\
    & +pl_{t,r}tl\_refund_{t,r,h}
  \end{alignedat}
  ,
  \label{eq:hh_budget}
 \end{equation}
```

AFTER:

```
  \begin{equation}
  \begin{alignedat}{1}
   \mathit{kh}_{t+1,r,h}+\mathit{pcl}_{t,r,h}\mathit{cl}_{t,r,h} = &
    \left(1+r_{t}\right)\mathit{kh}_{t,r,h}+\left(1-\mathit{tl}_{t,r,h}
    -\mathit{tfica}_{t,r,h}\right)\mathit{pl}_{t,r}\mathit{te}_{t,r,h}\\
    & +\mathit{pr\_ex\_agg}_{t,r}\mathit{kh\_ex}_{t,r,h}
    +\sum_{s}{\mathit{pres}_{t,r,s}\mathit{rese}_{t,r,s,h}}\\
    & +\mathit{pfx}_{t}\mathit{bopdef}_{t,r,h}+\mathit{cpi}_{t}\mathit{tran}_{t,r,h}\\
    & +\mathit{pl}_{t,r}\mathit{tl\_refund}_{t,r,h}
  \end{alignedat}
  ,
  \label{eq:hh_budgetb}
 \end{equation}
```

The following is the "before" and "after" of equation 23, interleaved to illustrate the difference made by employing \mathit{}:

$$kh_{t+1,r,h} + pcl_{t,r,h}cl_{t,r,h} = (1+r_t)\,kh_{t,r,h} + (1 - tl_{t,r,h} - tfica_{t,r,h})\,pl_{t,r}te_{t,r,h}$$

$$\begin{aligned} kh_{t+1,r,h} + pcl_{t,r,h}cl_{t,r,h} = {} & (1+r_t)\,kh_{t,r,h} + \left(1 - tl_{t,r,h} - tfica_{t,r,h}\right)pl_{t,r}te_{t,r,h} \\ & + pr_ex_agg_{t,r}kh_ex_{t,r,h} + \sum_s pres_{t,r,s}rese_{t,r,s,h} \\ & + pr_ex_agg_{t,r}kh_ex_{t,r,h} + \sum_s pres_{t,r,s}rese_{t,r,s,h} \\ & + pfx_t bopdef_{t,r,h} + cpi_t tran_{t,r,h} \\ & + pfx_t bopdef_{t,r,h} + cpi_t tran_{t,r,h} \\ & + pl_{t,r}tl_refund_{t,r,h} \\ & + pl_{t,r}tl_refund_{t,r,h} \end{aligned} ,$$

2. *Are the model structure and assumptions reasonable and consistent with economic theory?*

Yes, they seem reasonable to me.

3. *Are the inputs used in the model (e.g., elasticities, social accounting matrix) reasonable and reflective of the peer-reviewed literature?*

Yes, the inputs used seem broadly reasonable. Because of the open source nature of the framework, updating the baseline with new economic data or behavioral parameters is straightforward.

4. *Does the model produce intuitive and expected results?*

Yes, although I have not reviewed these as deeply as I would like.

5. *Each model run is subjected to a series of tests to verify that the solution represents an equilibrium. Additional tests are performed to verify that implicit parameters (e.g., labor supply elasticity) match their calibration targets. Are there other verification tests that should be incorporated into the model?*

I have no additional suggestions.

6. *While the most appropriate approach for modeling a policy will be regulation specific, is the general framework for capturing compliance requirements in the model reasonable? Are there other approaches that should be incorporated into the model?*

The framework illustrated is appropriate. Nonetheless, there will no doubt be cases in the future that call for novel approaches. The open-source nature of the model should ease the incorporation and vetting of any new methods.

7. *Is the outlined versioning framework transparent and reasonable? Do you have any specific suggestions for how to improve it?*

The versioning framework is transparent and reasonable. In my experience (granted, limited to CGE trade modeling), the approach laid out by EPA is much better documented than most models and consistent with software development best practices as I understand them. I applaud EPA's thoughtful and thorough approach.

8. *Are the criteria in EPA's memo for the types of model changes that warrant subsequent peer review reasonable?*

The criteria as laid out in the memo represent a good starting point for establishing when peer review is reasonable. As indicated in the memo, some cases will fall into a gray area (e.g., sectoral disaggregation for a specific application, changes to core elasticities). If there is a possible middle ground between a full SAB peer review such as this one and a purely internal review, I would suggest the EPA consider that in such cases, in particular in the early period after deployment of the SAGE model for rulemaking. Perhaps one or two external experts could be tapped to provide in effect a referee report on the change as implemented in these cases.

9. *Are the anticipated updates outlined in EPA's memo sensible next step improvements to the model and its parameterization?*

Yes, the approach laid out in the memo "Potential Near-Term Updates to the SAGE Model" is reasonable and recommended. As a trade modeler, I in particular recommend that some large open economy specification be implemented, especially in cases where regulation touches on energy-intensive, trade-exposed (EITE) industries (as noted in the memo). A full representation of the rest of the world, while desirable, is likely not necessary or tractable. The suggested approach of defining foreign export demand and import supply is a good compromise, especially if supply and demand are disaggregated across multiple regions active in the production and trade of the relevant goods.

10. *Does the SAB recommend additional near-term updates to the SAGE modeling framework or parameterization?*

I don't have any to recommend at this point. The framework presented and the sources for parameters used all are consistent with broadly accepted practices in the modeling field.

Comments from Dr. Mun Ho

1. Model documentation

This model documentation is one of the better ones around and I congratulate the SAGE team. Section 5 on modeling regulatory requirements is well done.

The model baseline is a balanced growth path, i.e., it is assumed that the exogenous variables are in the steady state from the first year (2016), with real values growing at $(1+\gamma+\omega)$. This should be made clear in the beginning rather than in section 3.4.

It would be helpful to have an Appendix with the full list of equations; right now there are few value balance equations where I can see how prices are multiplied by quantities. For example, right before eq. (2) there is mention of the pa, pn, pd, pfx prices, but they are not used in the equations that follow. They are only given implicitly much later in eqs. 88+.

In section 6 on using the Model, it might be good to note that there is a Public version without the data, and a Full version.

2. Model structure and assumptions, consistency with theory

2.1 Dynamic structure: Balanced path

The model assumes that the economy is in a balanced growth equilibrium, that all real exogenous variables grow at the steady state rate $(\gamma+\omega)$ in the base case. This is a common approach in macroeconomics using complicated stochastic models. This is an appropriate simplification for the analysis of many types of policies even using a non-stochastic model like SAGE. However, it might be good to think about when it is not appropriate. The actual economy is not in a steady state in that the population is still aging (growth of labor supply not equal to growth of population), and the government and trade deficits are not sustainable (either tax rates or spending have to change over the 2050 horizon). I have not calculated the "deficits" implied by the $tran_{t,r,h}$ variable, but would be very interested to see them (government consumption grows at $(\gamma+\omega)$ and tax rates stay at base year levels so *tran* should be a stable value, unlike actual CBO projections of the deficit, interest payments and official transfers). A policy that has a large impact on government revenues (e.g. carbon taxes) would play a big role in these deficit baselines. A more realistic base case path would have higher tax rates and thus bigger welfare effects of changing them due to, say, carbon revenues.

2.1.1 Balanced growth

I am not sure how others define a balanced growth equilibrium. In the textbooks we say that the steady state is when all relative prices stop changing and real quantities growth at the rate of effective labor. Such textbook models have a simple capital input that is reproducible capital. In SAGE there are resource inputs that are fixed in supply, and the prices *pres* diverge to infinity (e.g. pres(2061,enc,agr)=10.8 relative to cpi(2061) after starting at 1 in 2016). Are equations 46

and 47 valid in this framework? (I don't know the answer. Many models that have fixed resources are myopic.)

2.2 Productivity concepts and production function

Productivity in SAGE is represented by a Harrod-neutral growth in effective labor at rate ω.

Effective time endowment: $te_{t,r,h} = te0_{r,h}(1+\gamma+\omega)^t$

Labor: $\sum_h l_{t,r,h} = \sum_s ld_{t,r,s}$; $te_{t,r,h} = leis_{t,r,h} + l_{t,r,h}$

Production function: $kl_{t,r,s} = .[.(kd)^s + .(ld)^s]^{1/s}$

There are various definitions of productivity and I like to clarify them; how "labor productivity (LP)" is related to the ω in the above equations.

LP at the aggregate level, in the BLS definition, is GDP/hoursworked. In growth accounting we have the growth rate formula:

LP growth = capital deepening + labor quality growth + TFP growth

Or:

LP growth = capital stock deepening + cap composition(quality) + labor quality + TFP

(Labor quality is a compositional change index, a bigger share of total workers with higher skills over time. Capital quality is the compositional change with the changing ratio of equipment to structures.) If we use the concepts in Jorgenson, Ho and Samuels (2019)[1] then in the 1990-2014 sample period for the US, the 3 component contributions of LP are: 1.63 = 0.93 + 0.25 + 0.46. Or, when we use the 4-component formula, approximately: 1.63=0.70+0.23+0.25+0.46

If we think of a simple aggregate Cobb-Douglas function in a typical macro model, then the Hicks-neutral TFP of $\mu = 0.46\%$ per year is represented by:

$$Q_t = e^{\mu t} K_t^{\alpha} L_t^{1-\alpha}; \qquad L_t = e^{q_L t} N_t$$

In this formulation L is effective hours, and N is hours (number of workers); qL is the rate of growth of labor quality and K is the effective capital input. Converting this to the Harrod-neutral form gives:

[1] Jorgenson, Ho and Samuels. 2019. Education Attainment and the Revival of US Economic Growth, in Hulten and Ramey (eds) Education, Skills and Technical Change. NBER Studies in Income and Wealth Vol. 77.

$$Q_t = K_t^{\alpha}(e^{\omega t}L_t)^{1-\alpha}$$

where $\omega(1-\alpha) = \mu$, or $\omega = 0.46/(1-0.4) = 0.77$

Other authors do not distinguish the labor quality and capital quality effects and these are lumped into the TFP term. In this case, historically, TFP_b = 0.23+0.25+0.46=0.94.
And converting to the omega form:

$$\omega = 0.94/(1-0.4) = 1.6$$

Thus, *by coincidence*, this is equal to the average 0.016 cited by the SAGE document from the AEO projection of the *nonfarm private* labor productivity growth rate. Strictly speaking, the rate to use is the economy-wide LP, not just the private LP; which would be about 10% lower.

In the Jorgenson et al projections, we expect qL growth to fall a lot and TFP fall a little, and thus the outlook for TFP_b is more like 0.8 or lower, or an omega of 1.33. This together with capital growth and workforce growth we have almost 2.0% GDP growth.

The base case growth is SAGE is about 2.1% with the assumed $\omega = 0.016$ and workforce growth (equal to population growth) of 0.006. This workforce assumption is slightly higher than the 0.003 outlook for 2030s in the CBO and Jorgenson et al., both of which distinguishes population from workforce. It would be good to show in the base case description of the documentation, the capital growth, resource growth (0), labor growth and GDP growth in a Table for diagnostics.

2.3 Dynamic structure: 5-year time steps.

The 5-year time step is appropriate as a tradeoff point between accuracy and computational burden for long term policies such as carbon prices. Other regulations may have a shorter horizon. This model is designed with a distinction between extant capital and new capital, that is, built with short-run considerations very much in mind. It believe it would be worth it to have an uneven time spacing; e.g. every year for the first 5 years, then every 5 years. I realize few models do this, but Peter Wilcoxen is an expert on this.

2.4 Consumption

The consumption function is a CES function where the share parameters for energy is calibrated to AEO projections. The non-energy share parameters then undergo a small rescaling so that the shares for all 23 commodities add to 1. It would be better to project all share parameters based on models that have a richer set of income elasticities instead of assuming unit income elasticity. Right now, for example, the food share rises a bit due to the balancing of the declining energy shares. This is against the few widely accepted laws of empirical economics – Engel's Law. This is probably not a super important issue and I put it under item 9.2 for model improvements below.

2.5 Investment

The function allocating total investment to the various commodities is of the Leontief form. This is not reflective of past investment trends that show rising shares of, say, computer equipment. This is indeed a topic ignored by most models; but the cost of making it more flexible (e.g. CES) should be small. This will also allow the option to set an exogenous trend in the share parameter. The modeling of investment allocation is also discussed in the reference given in 9.2 below.

2.6 Government

Government industry is included as part of the Services industry. I would like to point out some implications of this simplification. 1. It means that the government capital input is mixed in with private capital input; this Kgov is unlike regular private capital that consist of depreciation and profits, it is an imputed value just based on depreciation of the vast stock of government capital. The rental rate of this giant Services sector is thus a mixed bag of market return and imputed depreciation and may give a wrong idea of the marginal cost of capital to services. This may not be an important simplification for the likely policy simulations; I am merely suggesting some more discussion.

2.7 Tax system

The attention paid to the tax system is impressive, distinguishing between average and marginal rates; using the CPS data and Taxsim model to calculate marginal tax rates. Section 3.2 of the documentation states that *tk* is made up from the corp tax rate from CBO and personal income tax data. It would be good to clarify if the resulting tax revenues match the total revenues given in the National Accounts. I think tax rates should be calibrated to replicate the NIPA revenues and it would be good if the *tran* variable in the base year can be related explicitly to items in the NIPA government accounts. Similarly, the production tax *ty* based on IMPLAN should be calibrated to produce the corresponding total revenues in the NIPA in the base year.

3. Model inputs reasonable and reflect literature?

3.1 General discussion

The discussion of functional forms and parameter choice and relation to literature is very well done. The attitude towards choosing parameter values reflected in the writeup is very appropriate.

3.2 Resource prices

The estimation of resource prices reflects work that I think is pioneering and not often discussed in other models. The document only cites Sue Wing (2001), I like to confirm that the SAGE team could not find any other CGE model discussion of such resource estimates?

3.3 Productivity parameters.

See above comment 2.2. I do not know how important this is, perhaps not at all for most EPA policies. It is a matter of a base case growth of 2.1 versus 1.9/1.8 cumulated over 40 years.

4. *Does the model produce intuitive and expected results?*

4.1 I reviewed Marten, Garbaccio and Wolverton (2019) comparison of general equilibrium costs with partial equilibrium estimates and find the results reasonable. (The paper is also very well written to explain the issues.)

4.2 One odd thing I found in the base case that would be good to clear up: pk(2016,r) are all close to 1.0 except for r={mat, esc} with 1.25.

5. *Additional verification tests.*

I think it would be useful to have more information about the base case, i.e. more figures and tables in section 3.4. E.g. a figure for GDP(t) and the C and I proportions; a figure in efficiency units showing convergence to steady state values; the relative prices of key prices (pk, pl, pres, py(agf), pf(cru), etc).

6. *While the most appropriate approach for modeling a policy will be regulation specific, is the general framework for capturing compliance requirements in the model reasonable? Are there other approaches that should be incorporated into the model?*

The two options for modeling regulations – productivity shock and explicit abatement technologies – are good and appropriate. The need to parameterize these functions point to the need for detailed abatement cost data. In the discussion of the results for the regulatory example in section 5, I think it would be instructive to talk about the capital losses – the change in pk in the regulated sector.

7. *Is the versioning framework transparent and reasonable*

Yes, it seems well thought-out.

8. *Are the criteria in EPA's memo for the types of model changes that warrant subsequent peer review reasonable?*

Yes

9. *Potential Updates to SAGE*

9.1 The idea to improve the representation of product taxes is excellent and I think should be prioritized. Specifically, the taxes on gasoline are now deemed to be a tax on the Retail industry and that is not an economically useful representation. Reorganizing it so that these revenues are attributed to the Refining sector would be more economically meaningful and relevant when considering further increases in fuel taxes.

9.2 Improving the Consumer demand system by using the CEX to estimate elasticities (by region and income groups). This is a great, if ambitious, project. I am impressed with the group's

knowledge of this literature. In the context let me note a just-finished review paper on this very topic "Modelling consumption and constructing long-term baselines in final demand" by Ho, Delzeit, Leblanc, Schuenemann and Weitzel (2019, written for GTAP-OECD conference on baselines).

Doing a model comparison exercise (i.e. estimating different models and then simulating SAGE with them) would be a big time commitment. This might be a topic for a broader modeling community cooperative exercise (huge positive externalities).

A much more relevant and easy improvement would be to calibrate the consumption shares using a model with richer income elasticities than the assumed 1.0 for all non-energy goods. These are done in other models discussed in Ho, Delzeit et al (2019).

9.3 Modeling the Rest-of-World. The suggestion to avoid a full-blown endogenous ROW but use a reduced form world demand and world supply curves is something I have great sympathy with. As the SAGE group notes, there are only a few regulations/policies where world trade is important are the large policies such as fuel taxes or carbon taxes, and these are not really EPA policies.

10. Additional near-term updates

10.1 Construction of an emissions inventory for SO2, NOx, PM and other main GHGs.

10.2 Changing function allocating total investment to commodities from current Leontief form to more flexible functions

Comments from Dr. David Montgomery

1. *Is the model documentation clear, accurate, and transparent? Do you have any specific suggestions for how to improve it?*

Yes, the documentation is reasonably clear and transparent and as accurate as I can ascertain without line by line comparison with sage.gms. I am particularly impressed by the discussion of the sources from which assumptions about elasticities and other parameters are derived, and that reasons are given for all the choices. The discussion of the variation in estimates across recent studies makes the range of possibilities clear, and good reasons are given for choices.

I am also pleased to see that explanations of the implications of parameter and of model structure choices are provided.

There are some points where I found explanations a little to terse, and some gaps.

a. The importance of changes in the prices of inputs to producing the investment good is mentioned several times, but I could not find a specific discussion of the production function for the investment good. Since new capital is malleable and assigned to sectors by sharing based on values in the SAM, I assume that there is just a single investment good not one differentiated by sector. From the balance conditions, I take it that the investment good is made up of output of each sector not otherwise assigned to government and household consumption or net exports. Given the importance attached to changes in the cost of the investment good for sectoral and dynamic impacts, it would be good to have a separate discussion of all this, if only to help readers like me find answers to these questions.

b. The assumption that resource industries have a fixed factor (land, resources in the ground) is valid and conventional. The documentation does not discuss whether the fixed factor varies over time to represent resource depletion, and appears to suggest that whatever exhaustion occurs is due to decreasing returns in the presence of the fixed factor. That is different from the way some other models calibrate for depletion, and possibly superior, and needs more discussion.

c. It is not entirely clear to me how the productivity shocks are implemented for regulations that are phased in over time.

2. *Are the model structure and assumptions reasonable and consistent with economic theory?*

Yes. The basic model structure and assumptions are consistent with economic theory and overall represent reasonable compromises given the limitations of data and the likelihood that the model will solve consistently.

I would like to give the discussion of the baseline a second reading, as I do not have it all clear in my head. My understanding is that you start with a balanced growth path – in particular the documentation states that physical capital in all sectors is forced to grow at the same rate. Then

the baseline is calibrated to reduce coal use in electricity, match the AEO energy efficiency improvement with AEEI embodied in capital, and other parts of the EIA forecast.

This is a standard practice. What might be desirable is to calibrate other sectoral growth rates to a forecast of secular change in the industry mix in the U.S. I am not sure I believe such forecasts, but the assumption of balanced growth will produce different magnitudes of impacts for sectoral policies than would the assumption that the affected industries will be growing or declining over time. Those considerations do come into some regulatory debates.

If I have become confused about how the baseline to be compared to the policy case is constructed, I apologize. It might help an inattentive reader like me to discuss more fully what variables are chosen for calibration and how the matching of the SAGE baseline to the external forecast is tested.

The treatment of the oil and gas sector is described very clearly, and the method by which a supply elasticity for crude oil is derived is admirable.

I am not quite so satisfied with the electricity sector. This is where experience suggests that the CES specification cannot adequately capture fuel choice for power generation. Substitution between coal and gas (primary energy) is good enough for the fossil energy input, but the lack of explicit treatment of energy inputs from nonfossil sources means that economic choices of such forms of energy has be captured in the elasticity of substitution between value added and energy. Thus investing in nuclear increases capital and labor inputs so that more energy can be produced with the same amount of fossil energy input. Ultimately, this will be unsatisfactory.

Likewise, treating motor vehicles and petroleum as substitutes in the CES nest can only be justified if you do not intend to analyze any regulations affecting motor vehicles or petroleum costs. There is a much more complex relationship there, and I think that both US-REP and NewERA have a nest that combines fuel, O&M, and vehicle services into a transportation aggregate that is consumed by the household. In this framework, an increase in fuel cost or vehicle cost will reduce transportation demand, and fuel economy can be improved by substituting vehicle services for fuel.

3. *Are the inputs used in the model (e.g., elasticities, social accounting matrix) reasonable and reflective of the peer-reviewed literature?*

Yes. The discussion is thorough and the values chosen are in the ranges I would expect. The effort to find specific empirical estimates for each substitution elasticity rather than some generic assumption is particularly admirable and revealed a number of studies I never had the resources to track down.

The use of Taxsim to compute marginal effective tax rates for the various taxes by sector or income group is very impressive and puts SAGE in a league of its own in its ability to address very interesting questions about tax interactions. I am curious whether the most recent version of

TAXSIM is used, because the effective tax rate I saw discussed seem high in light of the TCJA rate reductions.

4. *Does the model produce intuitive and expected results?*

Yes, for the examples provided. This is a question that needs to be answered on a case by case basis. In terms of identifying bugs in the basic model, I think that the results are intuitive and suggest that the code is working as expected. For any particular analysis, answering this question can be aided by producing more graphics showing key results for macro and industry variables, so that magnitudes and variations over time can be visualized. With a dynamic, forward looking model and policies whose impacts vary over time, there is always a possibility that the model agents are anticipating future costs and rearranging consumption and investment over time in ways that lead to impacts on consumption and EV that are not simple to explain.

5. *Each model run is subjected to a series of tests to verify that the solution represents an equilibrium. Additional tests are performed to verify that implicit parameters (e.g., labor supply elasticity) match their calibration targets. Are there other verification tests that should be incorporated into the model?*

This question requires some thought and I am not quite prepared to comment at this point. The calculations of implicit parameters are quite valuable and capture many of the things I would look for heuristically when reviewing results. This is a nice summary way of providing a basis for question 4, because the most effective use of intuition that I have found is whether the magnitudes of the impacts are congruent with the magnitude of the shock, and with each other.

6. *While the most appropriate approach for modeling a policy will be regulation specific, is the general framework for capturing compliance requirements in the model reasonable? Are there other approaches that should be incorporated into the model?*

The approaches of productivity shocks or production of an abatement good are in common use and provide a good general framework. The similarities and differences in results described in the documentation and the paper on the subject are what I would expect.

Including an abatement good into a constant returns production function implies decreasing returns to the original factors of production and to abatement. The magnitude of abatement cost relative to cost of production will determine how rapidly returns decrease, and even if air regulations are normally small relative to the economy of a whole they may be more significant for particular industries. I [could/could not] detect this in the stylized cases compared. To be useful in identifying industry impacts, more information is needed to create unique input-output coefficients for production of the abatement good.

Because of decreasing returns, calibration of the production function with an abatement good should take into account existing regulations. That is, the marginal cost of increasing use of the abatement good by a small amount will be strictly greater than zero if some amount is required

by pre-existing regulations. This suggests that there may be more than one abatement good, and the effect of each on the marginal cost of others will matter.

The productivity shock approach is much simpler, requiring only an engineering cost analysis adequate to estimate unit factor requirements and assign the shock to labor, capital, energy or materials. In the case of materials, assigning the shock to particular materials would require the same data as determining input requirements for production of an abatement good. The amount of substitutability among material inputs should have a strong effect on the equilibrium loss of output from a shock to the productivity of a single input. To maintain the simplicity of this approach and avoid variation in cost estimates based on parameter choice, it might be better to assume that the productivity shock applies to the materials nest and not to individual material inputs.

For either one to be useful, the structure of the SAGE model needs to incorporate the important mechanisms by which a regulation is expected to operate. Thus, a complete engineering study of a regulation will give the required labor, capital, materials and energy for construction and operation, so that if it is possible to map the target sources into industry codes, it should be possible to calculate the size of the shock relative to the size of the industry represented in the SAM. I get the impression that these shocks are implemented once for all in the first time period for which the model is solved. Since many regulations are phased in, it would be useful to have a capability to change the size of the shock over time.

It is not clear to me how existing regulations affecting a sector are included in the baseline. Since there are no structural representations of regulation in the model, I take it that compliance costs with, for example, current air regulations on powerplants are just in the SAM data for unit costs in that industry. Any decreasing returns to emission control or interactions with controls already required would have to be addressed in the engineering analysis.

One of the conveniences of the productivity shock approach is that it is not necessary to calibrate the model to mimic the results of an engineering analysis – the production function is simply adjusted to incorporate the engineering analysis directly.

Adding more structural detail to the model in key sectors subject to repeated regulation – such as electricity generation and motor vehicles – would have some benefits. It would make it possible to include more of the margins on which decisions are made in response to regulations, and therefore opportunity costs are incurred. This additional structure is often found in partial equilibrium models, for example those used to study the impact of emission standards on motor vehicles, and has been very useful to identify ways in which the fine detail of regulations leads to perverse behavioral changes.

The challenge in moving from an engineering analysis to a structural PE model to a GE model with some structure is that each broader model must be calibrated to the narrower model, rather than just taking the results of the narrower model to alter productivity.

7. *Is the outlined versioning framework transparent and reasonable? Do you have any specific suggestions for how to improve it?*

No comment – outside my experience.

8. *Are the criteria in EPA's memo for the types of model changes that warrant subsequent peer review reasonable?*

I see nothing wrong with them as general criteria. The comments on specific regulations may surface disputes about the validity of changes made in model parameters or structure for that particular RIA. When such disputes arise, the disputed changes could benefit from peer review either as part of the particular regulatory proceeding or when similar changes are made in the future.

9. *Are the anticipated updates outlined in EPA's memo sensible next step improvements to the model and its parameterization?*

I know relatively little about econometric estimation of different demand systems, and cannot comment on the relative merits of the different approaches. I do wonder whether the capacity to have income elasticities =/ 1 or estimated cross-elasticities can make much difference, given the relatively small magnitude of most regulatory impacts and the very gross categories into which consumer demand is divided. Relative price changes for those aggregates are not likely to be large in percentage terms nor are income effects, so that I would not expect much difference from the CES specification. Therefore, the Linz and Rutherford question of whether greater detail (i.e disaggregation) of demand will make a difference seems to me more productive. That is, if the impacts of a regulation are relatively concentrated on a particular consumer good within the broader categories, I would expect its marginal impact to increase more rapidly since it will be a considerable larger percentage of baseline cost.

I have a great deal more experience with SOE and other approaches to introducing trade into US models, and am confident that this improvement will pay dividends. I agree that embedding the U.S. model in a full GE model of global trade, though an elegant solution, would put excessive demand on EPA resources for relatively little increase in insight for typical air regulations. It is, on the other hand, hard to deal with bigger regulations like the Clean Power Plan without a global model.

I have found that the approach of adding an import supply curve to the model for commodities whose prices are affected by the regulation and for which the U.S. is a large buyer or seller on the world market is very useful and not complex. Especially with the assumption of period by period constant balance of payments, there is no need to add such supply curves for every commodity – just those that satisfy the conditions stated. EPA is quite correct that the result is likely to be shifting some of the costs of regulation to trading partners. The other interesting feature is that the decline in output of directly affected industries is likely to be larger due to import substitutions, but the offsetting effect due to the change in the value of the dollar/balance

of payments constraint is that other industries will gain output because of the opposite import substitution effect.

The idea of a single other country and producer is intriguing. It does raise questions about how the SAGE model should be closes for trade. If a change in capital flows and the merchandise trade deficit is allowed in the global model, care has to be taken to avoid end effects that push costs out beyond the horizon. This effect is the opposite of that which the assumption of period by period fixed balance of payments produces, which may exaggerate costs of rapid transitions in terms of consumption because of the inability to rearrange consumption through borrowing.

I see the point of taking excise and sales taxes out of the retail sector and assigning them individually to the commodities that are taxed. That should alter tax interaction effects and is worth exploring. I do wonder whether it will be possible to create a useful allocation of these taxes within the IMPLAN framework, and wonder if the newer public domain dataset could do a better job.

10. Does the SAB recommend additional near-term updates to the SAGE modeling framework or parameterization?

I would recommend working on more structural detail in household transportation and in electricity generation, within the CES framework.

These are important to identify the cross effects of changes in the costs of new vehicles, VMT, fuel cost on emissions, and without some such detail on household transportation, it will be impossible to account for take-back effects of policies such as greenhouse gas emission standards (e.g. fuel economy standards).

Likewise, with the structure for electricity production that I see in the documentation, it is very difficult for me to believe that SAGE will be able to produce any kind of reasonable results on how regulations for different sources of emissions from powerplants will affect fuel choice for generation or investment in renewables.

Comments from Dr. Sergey Paltsev

1. *Is the model documentation clear, accurate, and transparent? Do you have any specific suggestions for how to improve it?*

The model documentation of SAGE is clear and transparent to an experienced computable general equilibrium (CGE) modeler. I think that it would be useful to add to the model documentation a section targeted at non-CGE modelers that would explain the basic principles of CGE modeling and the dynamics represented in this particular version of the SAGE model.

In terms of accuracy, I was able to trace the correspondence of major structures and parameters to the documentation, but I have not assessed the accuracy of all details of the model.

The documentation does not specify time steps of the model and the model horizon. The model file *parameters.gms* provides a setting for a set *t* in 5-year steps from 2016 to 2061, but it is not clear from the documentation if the model can be run at different time intervals and for different time horizons.

The discount rate is a critical parameter for dynamic models and there is a substantial literature on a choice of discount rates (e.g., Stern vs Nordhaus). The model documentation in Section 3.4 provides the assumed rate of time preference (and therefore the discount rate), but it does not refer to any of the debates about the choice of the discount rate. If the authors believe that the debate on proper discounting is settled or not relevant for their purposes, it should be explained in the model documentation.

On a more pedantic side, the term "CGE" has never been introduced in the documentation (it appears for the first time on page 5 as CGE) and the model is stated to be an applied general equilibrium model (without clarifying that the authors treat AGE model being the same as CGE model). There are some passionate papers that make a distinction between AGE and CGE (e.g., Mitra-Kahn, Benjamin H. (2008) "Debunking the Myths of Computable General Equilibrium Models." Schwartz Center for Economic Policy Analysis, Working Paper 2008-1), therefore I would encourage a better description of a general approach.

2. *Are the model structure and assumptions reasonable and consistent with economic theory?*

The model follows a conventional way of representing profit-maximizing firms (producers), welfare maximizing households (consumers), subject to market clearance conditions where supply equals demand in all markets.

The closures of the model are explained and documented. I have a concern for the closure for balance of payment. It is exogenous, but the reasonable profile is not justified (running a deficit forever under an assumption of perfect foresight might be questionable).

It is not clear why only nine regions are represented in the current version of SAGE. Are there inherent limitations to represent more regions? Representing all states of the U.S.? If yes, these limitations should be described.

3. *Are the inputs used in the model (e.g., elasticities, social accounting matrix) reasonable and reflective of the peer reviewed literature?*

The inputs of the model are reasonable and reflective of the literature.

4. *Does the model produce intuitive and expected results?*

The model produces the results consistent with perfect foresight assumptions. However, in addition to an anecdotic evidence of limited foresight in many economic and political decisions, there are some empirical studies that also question this assumption (e.g., Li et al in Management Science (2014, 60, 9) evaluated air-travel industry and found that only 5.2% to 19.2% of population is strategic with perfect foresight). I would like to see more justification for the choice of the perfect foresight setting of the model.

5. *Each model run is subjected to a series of tests to verify that the solution represents an equilibrium. Additional tests are performed to verify that implicit parameters (e.g., labor supply elasticity) match their calibration targets. Are there other verification tests that should be incorporated into the model?*

A test to changing energy prices would be useful (e.g., WTI in 2016 was $43/bbl, in 2017 - $51/bbl, and in 2018 - $65/bbl). With historic price shocks, would the model reproduce historic regional production/consumption changes?

I would also like to see an exploration of sensitivity to a choice of the time intervals and model horizon (e.g., if the model runs at a shorter time interval (1 year instead of 5 year) and/or only up to 2031 (instead of 2061), would the results of the policy be the same overall (e.g., in terms of EV) or even the same in some particular year, let say in 2031 (e.g., for change in output)?).

6. *While the most appropriate approach for modeling a policy will be regulation specific, is the general framework for capturing compliance requirements in the model reasonable? Are there other approaches that should be incorporated into the model?*

The model documentation provides a discussion of modeling for both explicit compliance requirements and productivity shocks. Their representation rely on engineering estimates of compliance cost. These estimates should be obtained in separate studies and be reflective of regional/sectoral/temporal heterogeneities. Subject to availability of such estimates, the general framework for capturing compliance requirements is reasonable.

7. *Is the outlined versioning framework transparent and reasonable? Do you have any specific suggestions for how to improve it?*

The versioning framework outlined in the memo "Version Control Framework for the SAGE model" is transparent and reasonable. It is important to keep the versions used for particular rulemaking. As described in the memo, regulatory analysis undergoes several stages and it is important to keep the versions (and all used datasets) for intermediate stages and for the final rulemaking. The versioning for internal model development is also clearly outlined and reasonable.

8. *Are the criteria in EPA's memo for the types of model changes that warrant subsequent peer review reasonable?*

The major changes in the model structure and key assumptions would warrant additional peer review. The criteria for warranting subsequent peer review in the EPA's memo are reasonable.

9. *Are the anticipated updates outlined in EPA's memo sensible next step improvements to the model and its parameterization?*

The EPA memo list the following priorities for the SAGE model updates: 1) empirically informed consumer demand system, 2) large open economy specification, and 3) improved representation of production, sales, and excise taxes. I agree with the importance of these updates and the memo outlines the strategy for addressing these issues. Based on my crude assessment of the difficulties of these updates and their relative impacts, I would put them in the reverse order in terms of the priorities.

10. *Does the SAB recommend additional near-term updates to the SAGE modeling framework or parameterization?*

I would suggest adding to the near-term updates the following developments: working on supplementary physical accounts (energy, emissions) and alternative settings for representing expectations.

Comments from Dr. Thomas Rutherford

1. Functional forms

I would be somewhat cautious about taking on a demand system which is potentially non-convex at points on the price simplex far from the benchmark. At the same time, it seems important to have a demand system which can be calibrated to own-price and income-elasticities of demand. There are two approaches described in my paper with Bruno Lanz on GTAP9. The canonical GEMPACK model employs a constant difference of elasticities (CDE) demand system (Hanoch, 1975), which can be calibrated to own-price and income elasticities. GTAPINGAMS includes both the CDE system and a linear expenditure system (LES) representation, with code to parameterize the function to match income elasticities and average price elasticities.

There is a simple extension of the LES which can be calibrated to income and own-price elasticities of demand. I worked this up for GTAP9, but a referee didn't like it, so we took it out. I don't think that I'll publish it, but perhaps it could be helpful. I attach some lab notes (see Appendix 1).

2. Large economy closure.

It is a simple matter to introduce a downward sloping demand for US exports and an upward sloping supply of US imports. It is more difficult to specify the appropriate elasticities for these functions. We investigated this issue in a small open economy model for Chile several years ago. In our paper -- I believe it is ``Regional trading arrangements for Chile: Do the results differ with a dynamic model?", with ~David~Tarr. \textsl{Economie Internationale and Trade and Integration}~18:93--94, 2004 -- we demonstrated that by taking value shares and Armington elasticities from GTAP it is possible to incorporate the international trade responses of the multiregional trade model within a small open economy model. While this worked well for Chile, I'm not so sure that the same can be done for the US. It is, however, a simple matter to set up a multiregional GTAP model and see how it compares with a large open economy formulation based on the same data.

3. Incorporating Taxes

It is important to bear in mind that introducing taxes on sales should probably go hand-in-hand with attribution of retail margins. This could involve a lot of work. I would guess that this is not worth the effort at this point. What could be much more useful would be to include emissions coefficients in the production function. I understand that the model should be "driven" by engineering estimates of abatement costs, but if it is to be compared with conventional energy-economy-environment models, it should be able to compute permit prices and emission tax scenarios (at the very least). A key policy interest for EPA should involve the assessment of command-and-control as compared with tax instruments or quotas, and I don't think that the model as currently implemented can do this. I don't think that this should be so difficult -- attribute emissions to fuel use or to process.

I like the idea of hooking the process abatement technologies to engineering studies, but I think that this is something which could could incorporate endogenous adoption of specific measures. Abatement per unit output from a given mitigation measure has a cost and capacity. Taken together the abatement measures provide a staircase abatement cost curve.

4. Dynamics

I spent some time looking for a discussion of dynamic calibration in the model. It is not obvious how this is done -- the input-output table provides a snapshot of transactions in a given year, but there is typically a big discrepancy between the imputed capital stock and the requisite investment demand. I'd like to see how this is finessed in the baseline calculation. I'm also curious about how the partial-putty-clay model compares with a pure putty-putty model. Much is made of the transition dynamics provided by the partial putty-clay formulation. It would be helpful if the usefulness of the formulation could be demonstrated.

Comments from Dr. Ron Sands

1. *Is the model documentation clear, accurate, and transparent? Do you have any specific suggestions for how to improve it?*

   - Consider splitting model documentation into two parts: (1) Data and Parameters; and (2) Model Theory. Setting up the benchmark SAM deserves more discussion. Keeping these documents separate allows the authors to update these topics independently. One way to think about this is that there are two models: a calibration model and a simulation model.

   - Too much math – space could be better used to explain reasoning behind decisions made during model development. Only a few production or cost functions need to be presented in full detail. Beyond that, tree diagrams (e.g., Figure 2 and Figure 3) are more useful to the reader.

   - Model documentation (page 62) recommends the command line user interface, but many GAMS users are heavily invested in the GAMS Integrated Development Environment (IDE). Consider giving equal space in the documentation to running model code in either interface.

2. *Are the model structure and assumptions reasonable and consistent with economic theory?*

   - It would help to revisit the model's international trade closure. There are many options for handling international trade in a single-region model, as well as the choice of numeriare. I would be more comfortable with the U.S. as a region within a global model. How does the current structure of SAGE, with its price of foreign exchange, compare to a global model with real exchange rates? Is international trade closure different between the static and dynamic versions of SAGE?

3. *Are the inputs used in the model (e.g., elasticities, social accounting matrix) reasonable and reflective of the peer-reviewed literature?*

   - Use of a social accounting matrix (SAM) is essential to ensure that revenues match expenditures for all economic agents in the benchmark data set. Although this is standard practice for CGE modeling, it is generally not used in partial-equilibrium modeling or life-cycle analysis. The distinction becomes important when using a suite of models and weighing the relative contribution of various model types in a regulatory analysis. Some flexibility to modify the set of production sectors in SAGE would be helpful, to better match commodities in partial-equilibrium or life-cycle models.

   - A SAM ensures balance in terms of economic values, but there is also a need to maintain energy balance for analysis of environmental policy that involves use of fossil fuels. The SAGE model uses state-level energy data from the U.S. Energy Information

Administration (EIA), but energy balance tables from the International Energy Agency (IEA) may also be useful. In fact, the Global Trade Analysis Project (GTAP), as well as other large modeling teams, have worked through the details of reconciling energy balance tables with input-output tables to maintain energy balance within a SAM. This is not easy and requires expert judgement.

- I am somewhat concerned about the variability of substitution elasticities across production sectors in Table 8 (page 35 of model documentation). Trying to reconcile elasticity estimates across multiple studies with different production sectors and functional forms might not be the best strategy. Such wide variation across production sectors makes it difficult to explain model results. Further, it may be more informative to consider price elasticities of demand for consumers and producers, and income elasticities for consumers. The relationship between demand elasticities and substitution elasticities depends on functional form and level of nesting.

4. *Does the model produce intuitive and expected results?*

- It is too early in the review process to fully respond to this question. However, equivalent variation (EV) is one of the most important model outputs and I am puzzled to see the change in GDP to be of opposite sign as EV, in the basic example I was able to run.

5. *Each model run is subjected to a series of tests to verify that the solution represents an equilibrium. Additional tests are performed to verify that implicit parameters (e.g., labor supply elasticity) match their calibration targets. Are there other verification tests that should be incorporated into the model?*

- A diagnostic for energy balance would be helpful. One way to do this is to generate an energy balance table for all model time steps, as part of model output.

6. *While the most appropriate approach for modeling a policy will be regulation specific, is the general framework for capturing compliance requirements in the model reasonable? Are there other approaches that should be incorporated into the model?*

- The approach described in Figure 9 of SAGE documentation is very reasonable. It allows consistency between SAGE and engineering descriptions of abatement technologies, fully incorporates abatement costs into the cost of production, and distributes the cost of abatement across inputs needed for operating the abatement technology.

7. *Is the outlined versioning framework transparent and reasonable? Do you have any specific suggestions for how to improve it?*

   - This may be the strongest element of the SAGE modeling system. The versioning framework appears flexible enough to allow for major changes to model structure as needed.

8. *Are the criteria in EPA's memo for the types of model changes that warrant subsequent peer review reasonable?*

   - Consider a strategy to provide peer review of model code, and updates to documentation, at the same time major publications are released.

9. *Are the anticipated updates outlined in EPA's memo sensible next step improvements to the model and its parameterization?*

   - I agree that consumer demand is a good topic for near-term model development. There may be commodities where you do not want final consumption to scale with income, as would be the case with a nested CES consumer demand system. This is certainly true for agricultural commodities, but also true for energy-intensive services such as home heating. One possibility is to use the Linear Expenditure System (LES) as the top nest in a consumer demand system, and CES nests below that. In any case, the model should retain global consistency with curvature conditions, which limits the choice of functional form.

10. *Does the SAB recommend additional near-term updates to the SAGE modeling framework or parameterization?*

    - Keep in mind that the SAGE model will be one of many tools used in analysis of a given environmental policy or regulation. This may require that SAGE be very flexible in the number and type of production sectors, to better match an abatement technology or output from detailed life-cycle or engineering models. The option to collapse unnecessary detail, and expand where needed, would be helpful.

Comments from Dr. Dominique van der Mensbrugghe

1. *Is the model documentation clear, accurate, and transparent? Do you have any specific suggestions for how to improve it?*

Summary: Overall the documentation is quite impressive, well written and clear. I suspect a non-CGE audience, even of economists, might find it difficult to follow, but it is not intended as a primer for CGE modeling either. There were some excellent discussions, for example on the appropriate production nesting and the parameterization. Everyone has their own preferences for organizing the model description, I found this one to be somewhat haphazard. Here are a few recommendations:

- Describe first the comparative static model, leaving the dynamics till the end. So the budget constraint can be described vis-à-vis household savings, and then the savings/investment dynamic is described in a separate section.

- One way to organize the model description is using the standard circular flow paradigm: (1) production; (2) income allocation; (3) final demand; (4) domestic and international trade; (5) market equilibrium; and (6) closure. A suggested restructuring of the model code that follows this paradigm is provided as an annex. This version of the model code also keeps 'modules' together. This means that they can more readily be swapped out if changes are made.

- I think the dynamics needs a little bit more explanation, especially for someone like me not too familiar with perfect foresight. I would recommend starting with a simple framework and annual time steps. Then expand the simple framework to encompass the structural features of the model (e.g. multiple households) and the passage from annual time steps to multi-year time steps. This will make the stock/flow dynamics more transparent.

- The discussion in section (4) seems overly complicated. You've described most of the model equations earlier. The model is a set of N non-linear equations, that is solved using a Newton procedure. The model includes (presumably) a number of diagnostics that provide confidence in the resulting solution.

2. *Are the model structure and assumptions reasonable and consistent with economic theory?*

Below are a number of comments and recommendations—largely in random sequence.

- Drop the 'dtrd' and 'ftrd' distinction and use separate variable names for these.

- My preference is to keep base and purchasers' prices as separate variables—makes the expressions look more compact. This can be readily implemented using GAMS'

MACRO feature. In addition, I prefer all taxes to be valued relative to base prices and not to purchasers' or producer prices (see for example 'ty').

- Use more informative variable names. Try and avoid single letter names, typically used for sets (for example 'i').

- Could combine the 'extant' and 'new' nests in a single structure indexed by 'v'. This would add additional future flexibility, reduce code size and simplify the documentation.

- Note that the first use of the multiple nested energy structure was in the OECD's GREEN model (see van der Mensbrugghe, OECD 1994). GREEN was subsequently transferred to MIT and morphed into the EPPA model.

- Why not add a natural resource supply function? And/or supply shifters? No reason to assume that the tax on natural resource is the same as for capital, in fact I suspect that the treatment of natural resource income is quite distinct, particularly across states.

- Could use generic CES functions for final demand, and allow for zero elasticities.

- Not quite sure how to interpret 'extant' versus 'new' capital. In the 'GREEN' model (and subsequent derivatives such as ENVISAGE), 'Old' capital is allowed to increase over time. At the start of each new period, the 'fixed' 'Old' capital is equal to the sum of 'Old' and 'New' capital from the previous period (properly depreciated). Thus 'New' capital represents a relatively small share of total capital. In SAGE, capital installed in 2021 is still considered 'New' in 2061.

- Is it possible to have variable time steps?

- Equation (23) needs further discussion. (1) *kh* is called 'household savings invested in new capital stock', which can lead to interpretation issues. Given that *kh* is a stock, why use the word savings in this context? Given our earlier email traffic, I think the household budget constraint, particularly savings behavior, needs more exposition. It appears to me that there is an implicit assumption that all capital remuneration is saved, plus or minus some adjustment. I would like to see a formal development of the analytics of this (even in a simplified framework, i.e. where all other income is consolidated into a single variable).

- I have never worked with this type of model before, so I am confused about the numéraire choice. In recursive dynamic models, often the numéraire is fixed in each period (discounting would be done post-simulation), and typically all prices are simply

designated relative to base year prices. You can of course re-price future years post-simulation with SAGE as well, so this decision has no implications for model results. Is this something that could be further elaborated on?

- On the labor tax schedule, couldn't this be implemented with a single curve that allows for the marginal rate to differ from the average?

- For future use, you may want to describe the government budget constraint explicitly using a government savings variable. Your current closure assumes that this is set to 0. If you add this, you could simply exogenize government savings (or government savings wrt to nominal GDP) and then the lump sum transfers are endogenous to meet the fiscal target.

- I would be careful in the use the word 'clears' the goods market, for example for equation (36). The Armington price is a composite price and can be derived from the true equilibrium prices—which in the case of demand are *pd* the equilibrium price for domestic goods, *pn* the equilibrium price for national goods, and *pm* the equilibrium (though exogenous) price for imported goods.

- Is equation (41) a market clearance equation, or simply the motion equation for capital?

- Equation (42) is Walras' law in the benchmark year, where *pfx* is the numéraire and exogenous. As mentioned earlier, a further discussion on the numéraire and price choices might be merited.

- Is there a reason to maintain both unit costs and the respective price variable for the CES composites? The only bundle where they might deviate in practice is for output prices where any type of non-competitive assumption or deviation from CRTS would introduce a wedge between the unit cost of production and the producer price.

- Top of page (23). The imbalance between regions not only reflects investment flows, but also public expenditure flows—to the extent that the net public revenues in each region don't necessarily line up with public expenditures in each region.

- The income distribution aspects are a nice feature of the model—though I have some concerns about the dynamics since the structure of households is relatively static—no migration, no signification change in source of income, etc. Is EPA thinking of developing additional structural change as regards households?

- Similar reaction to the lack of labor mobility. This seems highly restrictive in a long-term model.

- Presumably EPA will be moving to the WiNDC database? Some of the advantages are clear, any drawbacks (e.g. household distribution). Note that WiNDC has a non-diagonal 'make' matrix. This would be relatively easy to implement in SAGE. Also on notation, I'm not fond of the use of 's' and 'ss' for sector. (In GTAP and other global models, 's' is often used for source country or region.) I would prefer 'i' for commodities, and 'a' for activities, and if you do implement a non-diagonal make matrix you will have to assume that the two are different.

3. *Are the inputs used in the model (e.g., elasticities, social accounting matrix) reasonable and reflective of the peer reviewed literature?*

- Not much to say about the incoming data. The fiscal side seems weak—there are essentially no sales taxes. Profit taxes are all incurred by the producer. GTAP differentiates between taxes on capital use (paid by the producer), and taxes paid on profits. The net effect (i.e. incidence is probably difficult to measure in a GE model). Resource payments also could use more scrutiny.

- Would probably avoid having tax rates in the database. GTAP has a history of providing all value flows in basic and purchasers' price from which tax rates can be readily derived. One advantage is that it makes aggregation trivial.

- I would minimize the amount of data massaging in the model code proper. For example, I would move natural resource payments to the input database.

- The literature survey on the elasticities is pretty thorough, even as thin as this literature is.

- The dynamic assumptions are straightforward, though it is not clear whether they lend themselves readily to assessing alternative baselines such as in the Shared Socio-Economic Pathways (SSPs) context.

- Could there be more heterogeneity, for example regarding population growth across regions/households, labor productivity across activities, etc.

- Why is labor productivity implemented through effective time endowment? Why not put it directly into the model's production nest? Does it matter?

- The dynamic energy adjustments could use more elaboration. It is not clear why the cost or preference parameters for non-energy demand need to be adjusted—one could just

adjust the efficiency of energy use—this, I believe, is the assumption in many other models. Perhaps I am misunderstanding something? Not that adjusting preference parameters couldn't be justified, it would simply need some intuition.

4. *Does the model produce intuitive and expected results?*

I haven't had much time to peruse the results in detail. There might be some additional useful indicators: ag + energy prices (wrt to CPI), energy intensity, agricultural and natural resource output growth, etc. Just one question about the dynamic simulations—do they change the benchmark data (i.e. the initial 2016 equilibrium)?

5. *Each model run is subjected to a series of tests to verify that the solution represents an equilibrium. Additional tests are performed to verify that implicit parameters (e.g., labor supply elasticity) match their calibration targets. Are there other verification tests that should be incorporated into the model?*

See comment on point 4. More broadly, one would one to compare the broad contours of the baseline scenario with existing baselines for example from the SSP database, other modeling efforts, etc.

6. *While the most appropriate approach for modeling a policy will be regulation specific, is the general framework for capturing compliance requirements in the model reasonable? Are there other approaches that should be incorporated into the model?*

The current framework is fine as a start. The only other obvious instrument would be some form of tax regime (simple tax, cap and trade, etc.) for example the SO_2 market.

7. *Is the outlined versioning framework transparent and reasonable? Do you have any specific suggestions for how to improve it?*

No other suggestions.

8. *Are the criteria in EPA's memo for the types of model changes that warrant subsequent peer review reasonable?*

No other suggestion. Hopefully, with the open source model, there will be uptake and a broader community that would be available to provide additional, if informal, review. This has been a strength of the GTAP network.

9. *Are the anticipated updates outlined in EPA's memo sensible next step improvements to the model and its parameterization?*

All three priorities seem reasonable and doable. From a specification perspective, this would be at most an afternoon's work for a skilled modeler. The trickier part will the parameterization and/or data collection and reconciliation (for example the tax system). On the consumer demand side it has been difficult so far to come up with a single specification that has optimal attributes:

parsimonious, easy to aggregate, good price and income elasticity behavior over time and across households. This has been the subject of the GTAP-led baseline project.

10. Does the SAB recommend additional near term updates to the SAGE modeling framework or parameterization?

Adding emissions to economic activity. May also want to develop more the agricultural, forestry and land-use sides—there are important regulatory issues in these areas.

How constraining is perfect foresight? Would a recursive dynamic model in parallel be useful and allow for greater flexibility—for example short time steps for the near-term and longer time steps for the distant future, more complex structures, greater heterogeneity… Could the two be coupled somehow?

Would suggest moving to a commodity/activity basis for the model. This would improve transparency and also allow for a relatively seamless move to the WiNDC database. It would also be a straightforward way to look at how changes in regulations may affect inherently heterogeneous activities—for example the power or steel sectors.[2] The model could incorporate a handful of different cost structures, that produce a single commodity. This feature is used extensively in integrated assessment models for modeling the power sector.

For the longer-term, the team may want to consider non-competitive behavior: IRTS, imperfect competition, less than perfect price adjustments (e.g. wages)…

Re-structured model code—a suggestion is provided in Appendix 2.

[2] Some of this is of course potentially picked up in the geographic location of activities.

Comments from Dr. Peter Wilcoxen

1. *Is the model documentation clear, accurate, and transparent? Do you have any specific suggestions for how to improve it?*

   - Both the standalone documents and the documentation embedded in the code for the model and its supporting programs are very well done. In general, both are thorough and clear. Model variables and parameters are named clearly as well. The directory structure is clear, and directories have readme files explaining their contents. Overall, very well done.

2. *Are the model structure and assumptions reasonable and consistent with economic theory?*

   - The treatment of the US as a small open economy is undesirable in terms of both flows of goods and financial capital. This should really be relaxed and it is good that it is on the list of potential near-term updates.

   - The model currently requires that both the government's fiscal budget and the nation's international trade accounts balance, although both are far from it in practice. A good first step would be to specify each exogenously. The fiscal deficit could be based on CBO projections.

   - Another important discrepancy between the model and the actual economy is the assumption about productivity growth in the baseline. Productivity growth is not uniform across sectors and there will be trend changes in relative prices in the baseline above and beyond the energy-related impacts currently addressed in the model. It would be good to move away from the current assumption of uniform labor-augmenting productivity growth across industries.

   - The definition of equivalent variation has an unconventional sign. The usual definition is the expenditure needed to get the new utility at the original prices less the original expenditure: $e(p_0, u_1) - e(p_0, u_0)$. Defined that way, a positive EV shows a gain in welfare. The definition in equation 134 appears to be the reverse, which is consistent with model results that produce positive EVs for regulations that should reduce welfare (since environmental benefits are omitted). In addition, it would be good to link the EV to wealth since the household's intertemporal expenditure should be consistent with its full wealth (including the imputed value of its leisure time).

3. *Are the inputs used in the model (e.g., elasticities, social accounting matrix) reasonable and reflective of the peer-reviewed literature?*

   - The model's parameterization is both an impressive achievement and its greatest weakness. SAGE includes a large number of very desirable features (e.g., regional production) but those usually include parameters that are very difficult to estimate. The model's current parameterization is impressive in that the authors have managed to find

or construct values for all of the required parameters. However, all of the parameters have been adapted from the literature, rather than being estimated specifically for SAGE, and many have been constructed by following rough guidelines, such as "the rule of two" used with trade elasticities, rather than via an explicit empirical strategy.

- Ideally the model's parameters should be estimated from time series data. Doing so would provide information about the precision of the model's parameters that is currently missing (e.g., no standard errors are provided). Moreover, it would provide a foundation for future calculations of the confidence intervals for modeling results. Probabilistic uncertainty analysis is recommended by OMB guidelines and EPA should plan ahead toward being able to carry out that kind of analysis.

- A good example is the differentiation between regional, national and international products. In principle, that can improve analysis of a policy's regional impacts. However, the quality of that regionalization will depend very heavily on the quality of the substitution elasticities between products from different origins. Without very solid parameterization, the model will produce spurious detail: e.g., apparent differences between regions that would not actually be statistically significant.

- A related drawback of the calibration approach used for SAGE is that it builds idiosyncrasies of the year of the SAM permanently into the structure of the model. That is, if δ_{ij} is the true share of input i into good j, the observed value at time will be d_{ijt}, which will include a disturbance ϵ_{ijt}: $d_{ijt} = \delta_{ij} + \epsilon_{ijt}$. Using a single SAM makes it impossible to know how large the error terms are, and it builds their values in a particular year into the model's coefficients.

- The empirically-estimated consumer demand system proposed as a near-term extension would be a very good improvement. It should be top priority, and ideally it should be the start of an extensive program to strengthen the model's empirical basis.

4. *Does the model produce intuitive and expected results?*

   - The results presented seem reasonable but I have not had time to run additional simulations to test that in detail.

5. *Each model run is subjected to a series of tests to verify that the solution represents an equilibrium. Additional tests are performed to verify that implicit parameters (e.g., labor supply elasticity) match their calibration targets. Are there other verification tests that should be incorporated into the model?*

   - A good test for each build of the model (but not each individual simulation) is to check that it is appropriately homogeneous in the numeraire. For example, change the exogenous numeraire price from 1 to 2 and verify that all price and value variables double but no quantity variables change.

6. *While the most appropriate approach for modeling a policy will be regulation specific, is the general framework for capturing compliance requirements in the model reasonable? Are there other approaches that should be incorporated into the model?*

   - The approaches outlined in the documentation are good and provide a great deal of flexibility. However, it would be valuable to develop an extended version of the explicit compliance approach where factor demands for abatement activities are linked to specific inputs rather than to the overall level of output. As it stands, the demand for specific abatement inputs are separable from the demands for productive inputs. That would, for example, make it difficult to model the impact of fuel switching between coal and gas within electric power generation. The current structure would not be able to endogenously capture the reduction in scrubbing that would occur.

7. *Is the outlined versioning framework transparent and reasonable? Do you have any specific suggestions for how to improve it?*

   - A key goal for the project is to build a modeling framework that can be adapted for different regulatory needs. As a result, it seems likely that there will be a number of long-lived variants: e.g., one with an extended treatment of electricity generation, another with more detail in motor vehicles, or a third with modeling of benefits. The current scheme appears to anticipate calling these branches SAGE #.#.#-rule_something but a better approach would be to name the major branches by their core features. E.g., a model with a more detailed electric sector would be SAGE-electricity-#.#.#. This would make the range and features of the variants clearer, especially to people outside EPA, and the versions used for particular rules and papers could still be indicated with tagging.

   - Also, the build process of the model seems to envision that new data would be drawn at build time from various data sources, such as the CPS. That raises a serious complication for the versioning scheme. It should be possible for a user to get a snapshot of both the code and data for a particular version of the model. That is obviously at odds with data being downloaded on the fly. This will eventually be very important when the Agency needs to interact with outside groups running the model: quickly and unambiguously tying down exactly which inputs are being used will be important in evaluating differences in results. To address this I would suggest including each version's fully-built input data in the main repository and using a separate repository for code used to build the data. That would improve the integrity of the naming convention (i.e., that someone running SAGE-branch-#.#.# will be using a known version of the code and data) but still preserve the open-source nature of the build process for users who need it.

8. *Are the criteria in EPA's memo for the types of model changes that warrant subsequent peer review reasonable?*

   - I agree with NCEE that it's hard to specify the exact criteria in advance. It certainly makes sense to have major revisions peer reviewed, but it might also be useful to have significant components reviewed separately. I'd lean against establishing that minor

revisions other than feature branches be peer-reviewed: doing so would impede development and use of the model by making it cumbersome to use the middle tier of the versioning scheme.

- As a concrete example, suppose the Agency implements the consumer-side change it proposes as a near-term revision. If that were the only change from 1.2.0, it would make sense to peer-review the module but not the whole model (since the rest would have just gone through this review). Because it's a substantial change that would likely be used in all subsequent versions, it would make sense to increment the major version to 2. In contrast, if the Agency moves away from a balanced-growth baseline, that should be a minor revision (e.g., 1.3.0) but would not, on its own, be a natural candidate for peer review.

9. *Are the anticipated updates outlined in EPA's memo sensible next step improvements to the model and its parameterization?*

- The proposed empirically-estimated demand system is a very good step and should be a high priority. As noted above, it would be great if this were the beginning of a long-term process of many more of the model's components.

- Moving to a large, open-economy approach is also a good step, and the plan to begin by adding demand and supply functions for US exports and imports is appropriate. It should be noted, however, that the US is also large in world capital markets and rarely runs a balanced current account. Future work should move toward including endogenous capital flows.

- Improving the treatment of taxes is also worthwhile.

10. *Does the SAB recommend additional near-term updates to the SAGE modeling framework or parameterization?*

- Moving away from a balanced growth baseline path should be a high priority. It should rank above improving the treatment of taxes as it is a larger discrepancy between the model and the actual economy.

- In general, moving toward stronger empirical parameterization is a high priority.

An Own-Price and Income Flexible Nested CES Demand System

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1 Data and Parameters

1.1 Benchmark data (calibration inputs)

θ_i benchmark value share (and reference demand) for the ith good, measured in Harberger units with benchmark prices of all goods are unity,

η_i income elasticity of demand for good i

$$\eta_i \equiv \frac{\partial x_i}{\partial Y}\frac{1}{x_i}\bigg|_{p_i=1,Y=1}$$

ϵ_i own-price elasticity of demand for good I

$$\epsilon_i \equiv \frac{\partial x_i}{\partial p_i}\frac{1}{x_i}\bigg|_{p_i=1,Y=1}$$

1.2 Income and prices

p_i price of the ith market good, $\bar{p}_i = 1$

Y consumer income (benchmark value unity).

1.3 Endogenous (calibrated) parameters

s_i subsistence demand for good i

a_i necessary (price inelastic) demand for good i

b_i marginal (price elastic) demand for good i

At the benchmark with $p_i = 1$ and $b_i = \bar{b}_i$:

$$\theta_i = a_i + \bar{b}_i + s_i$$

1.4 Benchmark shares (computed parameters)

$\bar{M}$ non-subsistence expenditure, $= 1 - \sum_i s_i = \sum_i (a_i + b_i)$

α_i value share of necessary input $= a_i / \bar{M}$

β value share of the marginal inputs $= (\sum_i b_i)/\bar{M}$, hence

$$\sum_i \alpha_i + \beta = 1$$

γ_i value share of the ith marginal input $= b_i/(\beta \bar{M})$, and

$$\sum_i \gamma_i = 1$$

1.5 Free Parameters

The elasticity structure for marginal demand is given exogenously. For local calibration, this elasticity is "free", but it's value could be estimated if the function were based on econmetric methods.

The unit cost function ($c(p)$) only needs to be linearly homogeneous in prices, and it thus provides a means of controlling cross-price elasticities if a more elaborate

demand system were desired.

σ elasticity of substitution among marginal goods

1.6 Shadow prices

The cost function for super-numerary consumption (c) is calibrated to unity in the benchmark

$$c = \sum_i \alpha_i p_i + \beta \pi(p)$$

where

$$\pi(p) = \left(\sum_i \gamma_i p_i^{1-\sigma} \right)^{1/(1-\sigma)}$$

An index of supernumerary expenditure is given by:

$$\phi(Y) = \frac{Y - \sum_i p_i s_i}{\bar{M} c}$$

2 The Demand Function

The demand for commodity i is:

$$x_i(p, Y) = s_i + \left[a_i + b_i \left(\frac{\pi(p)}{p_i} \right)^{\sigma} \right] \phi(Y)$$

Note that the function is *non-separable*, as the price of good i enters in both the Leontief and CES nests. The unit cost function is portrayed graphically in Figure **??**.

3 Price Elasticities

$$\epsilon_i = \frac{\partial x_i}{\partial p_i} \frac{1}{x_i} \bigg|_{p_i=1, Y=1}$$

Hence:

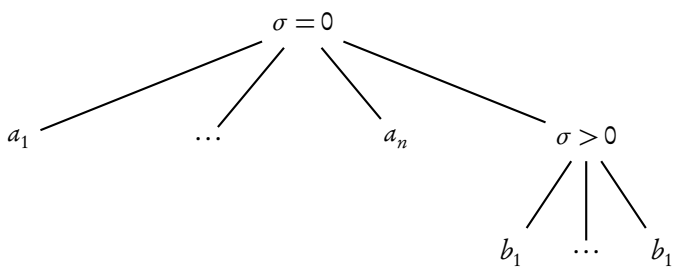

Figure 1: Substitution Structure in Supernumerary Demand

$$\frac{\partial x_i}{\partial p_i} = (a_i + b_i)\frac{\partial \phi}{\partial p_i} + \sigma b_i \left(\frac{\partial \pi(p)}{\partial p_i} - 1 \right)$$

Doing some calculus, we find:

$$\frac{\partial \phi}{\partial p_i} = \frac{-s_i}{\bar{M}} - \frac{\partial c}{\partial p_i},$$

$$\frac{\partial c}{\partial p_i} = \alpha_i + \beta\gamma_i,$$

$$\frac{\partial \pi(p)}{\partial p_i} = \gamma_i,$$

and

$$\begin{aligned}
\frac{\partial x_i}{\partial p_i} &= -(a_i + b_i)\frac{s_i}{\bar{M}} - (a_i + b_i)(\alpha_i + \beta\gamma_i) + b_i\sigma(\gamma_i - 1) \\
&= b_i\sigma(\gamma_i - 1) - s_i(\alpha_i + \beta\gamma_i) - (a_i + b_i)(\alpha + \beta\gamma_i) \\
&= -b_i\sigma(\gamma_i - 1)b_i - (\alpha_i + \beta\gamma_i)(s_i + a_i + b_i) \\
&= b_i\sigma(\gamma_i - 1) - \theta_i(\alpha_i + \beta\gamma_i)
\end{aligned}$$

Hence:

$$\epsilon_i = \frac{b_i \sigma(\gamma_i - 1) - \theta_i(\alpha_i + \beta\gamma_i)}{\theta_i} \tag{1}$$

4 Income Effects

So far as income effects, we have:

$$\begin{aligned} \eta_i &= \frac{\mathrm{dx_i}}{\mathrm{dY}} \frac{Y}{x_i} \\ &= (a_i + b_i) \frac{\mathrm{d}\phi}{\mathrm{dY}} \frac{1}{\theta_i} \end{aligned}$$

Hence:

$$\eta_i = \frac{a_i + b_i}{\bar{M}\theta_i} = \frac{\theta_i - s_i}{\bar{M}\theta_i} \tag{2}$$

5 Calibration

Given $\bar{M}$, we can solve for subsistence demand from equation (**??**):

$$s_i = \theta_i(1 - \bar{M}\eta_i).$$

Choosing $\bar{M}$ such that $s_i \geq 0 \quad \forall i$ we have:

$$\bar{M} = \frac{1}{\max_i \eta_i}.$$

We then solve the following system of equations to determine β and γ_i:

$$\min_{\beta, \gamma_i} \sigma$$

subject to

$$\theta_i(\epsilon_i + \eta_i\theta_i) = \sigma\beta\bar{m}\gamma_i(\gamma_i - 1) \quad \forall i$$

$$\sum_i \gamma_i = 1$$

$$\gamma_i\beta\bar{m} \leq \theta_i \quad \forall i$$

$$0 \leq \sigma$$

$$0 \leq \beta \leq 1$$

$$0 \leq \gamma_i \leq 1 \quad \forall i$$

GAMS Code for Calibration of GTAP 8.1

```
$title  Calibrate GTAP8 Using NNCES to Match Income and Own Price Elasticities

$if not set ds      $set ds      g20
$if not set datadir $set datadir .\
$include gtap8data

set     info    Information about this calibration /
        ds      "%ds%",
        datadir "%datadir%",
        workdir "%gams.workdir%"
        date    "%system.date%"
        time    "%system.time%" /;

parameter       vafm(i,r)       Aggregate final demand,
                theta           Value share in final demand,
                mbar(r)         Aggregate non-subsistance demand;

vafm(i,r) = vdfm(i,"c",r)*(1+rtfd0(i,"c",r))+vifm(i,"c",r)*(1+rtfi0(i,"c",r));
theta(i,r) = vafm(i,r) / (vom("c",r)*(1-rto("c",r)));
abort$sum(r, round(abs(1-sum(i,theta(i,r))),5)) "Shares do not add up.";

mbar(r) = 1/smax(i$eta(i,r), eta(i,r));

parameter       sc(i,r)         Subsistence consumption demand,
                sigma_(r)       Elasticity of substitution in marginal consumption;

*       Convert subsistence, necessary and marginal consumption levels to
*       benchmark levels:

alias (i,j);
sc(i,r) = theta(i,r)*(1-mbar(r)*eta(i,r)) * sum(j,vafm(j,r));

parameter       mbarchk Cross check on MBAR;
mbarchk(r) = mbar(r) - (1 - sum(i,sc(i,r))/sum(i,vafm(i,r)));
display mbarchk;

variable        GAMMA(i,r)      Marginal demand share,
                BETA(r)         Value share of marginal demand in supernumerary demand
                SIGMA(r)        Elasticity of substitution
                OBJ             Objective function;

equations       objdef, elasticity, simplex, addup;

objdef..        OBJ =e= sum(r, SIGMA(r));

*       EPSILON is the compensated elasticity of demand.  We equate this
*       to the compensated CES demand elasticity:

elasticity(i,r)$theta(i,r)..
                theta(i,r)*epsilon(i,r) =e=
                        SIGMA(r) * GAMMA(i,r) * BETA(r) * mbar(r) * (GAMMA(i,r)- 1);

simplex(r)..            sum(i, GAMMA(i,r)) =e= 1;
```

```
addup(i,r)$theta(i,r)..            GAMMA(i,r)*BETA(r)*mbar(r) =l= theta(i,r)*mbar(r)*eta(i,r);

model calib /objdef, elasticity, simplex, addup/;

SIGMA.LO(r) = 0.01; SIGMA.UP(r) = 20; SIGMA.L(r) = 1;
BETA.LO(r) = 0; BETA.UP(r) = 1; BETA.L(r) = 0.5;
GAMMA.LO(i,r) = 0; GAMMA.L(i,r) = 0.5; GAMMA.UP(i,r) = 1;
GAMMA.FX(i,r)$(theta(i,r) = 0) = 0;

solve calib using nlp minimizing OBJ;

parameter       nc(i,r)         Necessary consumption demand,
                mc(i,r)         Marginal consumption demand;

mc(i,r) = GAMMA.L(i,r) * BETA.L(r) * sum(j,vafm(j,r)-sc(j,r));
nc(i,r) = max(0,vafm(i,r)-sc(i,r)-mc(i,r));

sigma_(r) = SIGMA.L(r);

execute_unload 'calib.gdx',info, i,r, theta,epsilon,eta, sc, nc, mc, sigma_=sigma;
```

GAMS/MPSGE Code Verifies Calibration of GTAP 8.1

```
$title  Read the Calibrated Dataset and Verify Elasticities

set     i(*)            Goods
        r(*)            Regions

parameter       nc(i,r)         Necessary consumption demand,
                mc(i,r)         Marginal consumption demand,
                sc(i,r)         Subsistence consumption demand,
                theta(i,r)      Value share in final demand,
                sigma(r)        Elasticity of substitution in marginal consumption,
                epsilon(i,r)    Price elasticity of demand
                eta(i,r)        Income elasticity of demand;

$gdxin 'calib.gdx'
$loaddc i r  theta epsilon eta  sc  nc  mc  sigma

parameter       c0(i,r)         Aggregate non-subsistence demand,
                u0(r)           Non-subsistence expenditure
                gdp(r)          Base year gdp;

c0(i,r) = mc(i,r) + nc(i,r) + sc(i,r);
u0(r) = sum(i, mc(i,r) + nc(i,r));
gdp(r) = sum(i, c0(i,r));

alias (i,j);

parameter       thetachk;
thetachk(i,r) = round(c0(i,r)/sum(j,c0(j,r)) - theta(i,r), 5);
display thetachk;
```

```
parameter       mbar, mbarchk;
mbar(r) = 1/smax(i$eta(i,r), eta(i,r));
mbarchk(r) = mbar(r) - u0(r)/gdp(r);
display mbarchk;

parameter scchk;
scchk(i,r) = round(sc(i,r) - theta(i,r) * (1-mbar(r)*eta(i,r)) * gdp(r),5);
display scchk;

parameter gamma(i,r);
gamma(i,r) = mc(i,r)/sum(j,mc(j,r));


set             cc(i,r)         Consumption goods in the model,
                rav(r)          Representative agents in the model;

parameter       pindex(i,r)     Price index,
                gdpindex(r)     GDP index;

pindex(i,r) = 1;
gdpindex(r) = 1;


$ontext
$model:chkcalib

$sectors:
        U(r)$rav(r)
        C(i,r)$cc(i,r)  ! Supernumerary consumption demand

$commodities:
        PU(r)$rav(r)
        P(i,r)$cc(i,r)  ! Price index
        PM              ! GDP price index

$consumer:
        RA(r)$rav(r)

$prod:C(i,r)$cc(i,r)
        o:P(i,r)        q:c0(i,r)
        i:PM            q:(c0(i,r)*pindex(i,r))

$prod:U(r)$rav(r)  s:0  mc:sigma(r)
        o:PU(r)         q:u0(r)
        i:P(i,r)        q:nc(i,r)
        i:P(i,r)        q:mc(i,r)       mc:

$demand:RA(r)$rav(r)
        e:P(i,r)        q:(-sc(i,r))
        e:PM            q:(gdp(r)*gdpindex(r))
        d:PU(r)

$offtext
$sysinclude mpsgeset chkcalib

rav(r) = yes;
```

```
cc(i,r)$c0(i,r) = yes;

CHKCALIB.ITERLIM = 0;
$include CHKCALIB.GEN
SOLVE chkcalib USING MCP;

CHKCALIB.ITERLIM = 10000;
$include CHKCALIB.GEN
SOLVE chkcalib USING MCP;

parameter cref;
cref(i,r) = C.L(i,r);

parameter       elaschk(i,r,*)          Cross check on elasticities;

alias (r,rr), (i,ii);
loop(rr$(ord(rr)<=5),

*       First, check the income elasticity of demand:

        rav(r) = yes$sameas(r,rr);
        cc(i,r)$c0(i,r) = yes$rav(r);
        gdpindex(rr) = 1.001;

        CHKCALIB.ITERLIM = 10000;
$include CHKCALIB.GEN
        SOLVE chkcalib USING MCP;

        loop(i$round(theta(i,rr),3),
          elaschk(cc(i,rr),"eta") = eta(i,rr);
          elaschk(cc(i,rr),"eta0") = (C.L(i,rr)/cref(i,rr)-1)/0.001;
        );
        gdpindex(rr) = 1;

*       Second, check price elasticities of demand:

        loop(ii$(cc(ii,rr) and round(theta(ii,rr),3)),

          pindex(ii,rr) = 1.001;

$include CHKCALIB.GEN
          SOLVE chkcalib USING MCP;

          elaschk(cc(ii,rr),"theta")    = theta(ii,rr);
          elaschk(cc(ii,rr),"gamma")    = gamma(ii,rr);
          elaschk(cc(ii,rr),"epsilon")  = epsilon(ii,rr) - eta(ii,rr)*theta(ii,rr);
          elaschk(cc(ii,rr),"epsilon0") = (C.L(ii,rr)/cref(ii,rr)-1)/0.001;

          pindex(ii,rr) = 1;

        );

        rav(r) = yes;
);

option elaschk:2:2:1;
display elaschk;
```

---- 645 PARAMETER elaschk Cross check on elasticities

	eta	eta0	theta	gamma	epsilon	epsilon0
isr.CHN	1.05	1.05	0.02	0.01	-0.38	-0.38
isr.JPN	1.02	1.02	0.03	0.03	-0.79	-0.79
isr.KOR	1.05	1.05	0.03	0.03	-0.69	-0.69
isr.IDN	1.07	1.07	5.176752E-3	3.863311E-3	-0.29	-0.29
isr.IND	1.08	1.08	5.531530E-3	4.605816E-3	-0.25	-0.25
obs.CHN	1.67	1.67	0.02	0.03	-0.57	-0.57
obs.JPN	1.08	1.08	0.03	0.02	-0.82	-0.82
obs.KOR	1.16	1.16	0.02	0.02	-0.74	-0.74
obs.IDN	1.66	1.66	0.01	0.02	-0.47	-0.47
obs.IND	1.89	1.89	0.02	0.03	-0.47	-0.47

Suggested Code Restructuring from Dominique van der Mensbrugghe

```
* hicksian demand equation - abatement with extant capital
hde_abate_ex(t,r,s,"extant")$(abate0(t,r,s,"extant")).. 
   abate(t,r,s,"extant") =e= y_ex(t,r,s);

* hicksian demand equation - intermediates for use with extant capital
hde_id_ex(t,r,ss,s)$(extant_share(r,s) and id0(r,ss,s)).. 
   id_ex(t,r,ss,s) =e= prod_ind(t,r,ss,s,"extant")*y_ex(t,r,s);

* hicksian demand equation - labor for use with extant capital
hde_ld_ex(t,r,s)$(extant_share(r,s) and ld0(r,s)).. 
   ld_ex(t,r,s) =e= prod_ind(t,r,"l",s,"extant")*y_ex(t,r,s);

* hicksian demand equation - sector specific extant capital
hde_kd_ex(t,r,s)$(extant_share(r,s) and kd0(r,s)).. 
   kd_ex(t,r,s) =e= prod_ind(t,r,"k",s,"extant")*y_ex(t,r,s);

* hicksian demand equation - fixed factor resources with extant capital
hde_res_ex(t,r,s)$(extant_share(r,s) and res0(r,s)).. 
   res_ex(t,r,s) =e= y_ex(t,r,s);

* unit cost equation - output with extant capital
uce_y_ex(t,r,s)$(extant_share(r,s) and y0(r,s)).. 
   uc_y_ex(t,r,s)
      =e= (sum(ss, cs_y_ex(t,r,ss,s)*prod_ind(t,r,ss,s,"extant")*pa(t,r,ss))
       +   cs_y_ex(t,r,"l",s)*prod_ind(t,r,"l",s,"extant")
       *    (1+tp(t,r))*pl(t,r)/(1+tp0(r))
       +   cs_y_ex(t,r,"k",s)*prod_ind(t,r,"k",s,"extant")*(1+tk(t,r))
       *    pr_ex(t,r,s)/((1+tk0(r))*pr0)
       +   cs_y_ex(t,r,"res",s)*(1+tk(t,r))*pres(t,r,s)/(1+tk0(r))
       +   cs_y_ex(t,r,"abate",s)*pabate(t,r,s,"extant"))
       /    (1-cs_y_ex(t,r,"abate",s))
       *  (1-ty0(r,s)) ;

* zero profit - output with extant capital
zp_y_ex(t,r,s)$(extant_share(r,s) and y0(r,s)).. 
  uc_y_ex(t,r,s) =e= py(t,r,s)*(1-ty(t,r,s));

*  ---------------------------------------------------------------------------
*
*     Production module -- new capital
*
*  ---------------------------------------------------------------------------

* hicksian demand equation - abatement with new capital
hde_abate(t,r,s,"new")$(abate0(t,r,s,"new")).. 
   abate(t,r,s,"new") =e= y(t,r,s) ;

* hicksian demand equation - resource(r)-capital(k)-labor(l)-energy(e)-
* materials(m) bundle
hde_rklem(t,r,s)$(y0(r,s)).. 
  rklem(t,r,s) =e= y(t,r,s) ;

* unit cost equation - output with abatement using new capital
uce_y(t,r,s)$(y0(r,s)).. 
   uc_y(t,r,s) =e= (cs_y(t,r,"abate",s)/cs_y(t,r,"rklem",s)*pabate(t,r,s,"new")
                +    prklem(t,r,s))*(1-ty0(r,s)) ;

* zero profit - output with new capital
zp_y(t,r,s)$(y0(r,s) and not (tfirst(t) and extant_share(r,s) eq 1)).. 
   uc_y(t,r,s) =e= py(t,r,s)*(1-ty(t,r,s)) ;
```

```
* hicksian demand equation - fixed factor resource
hde_res(t,r,s)$(res0(r,s))..
  res(t,r,s) =e= y(t,r,s)
              *  (uc_y(t,r,s)/(1-ty0(r,s))*(1+tk0(r))/((1+tk(t,r))*pres(t,r,s)))
              **se_rklem(t,r,s) ;

* hicksian demand equation - capital(k)-labor(l)-energy(e)-materials(m) bundle
hde_klem(t,r,s)$(y0(r,s))..
    klem(t,r,s) =e= y(t,r,s)
                 *  (uc_y(t,r,s)/(1-ty0(r,s))/pklem(t,r,s))
                 **se_rklem(t,r,s);

* unit cost equation - resource(r)-capital(k)-labor(l)-energy(e)-materials(m)
* bundle
uce_rklem(t,r,s)$(y0(r,s))..
   uc_rklem(t,r,s) =e=
         (cs_rklem(t,r,"res", s)*((1+tk(t,r))*pres(t,r,s)/(1+tk0(r)))
      **(1-se_rklem(t,r,s))
   +     cs_rklem(t,r,"klem",s)*pklem(t,r,s)**(1-se_rklem(t,r,s)))
      **(1/(1-se_rklem(t,r,s))) ;

* zero profit - resource(r)-capital(k)-labor(l)-energy(e)-materials(m) bundle
zp_rklem(t,r,s)$(y0(r,s) and not (tfirst(t) and extant_share(r,s) eq 1))..
   uc_rklem(t,r,s) =e= prklem(t,r,s);

* hicksian demand equation - capital(k)-labor(l)-energy(e) bundle
hde_kle(t,r,s)$(y0(r,s))..
   kle(t,r,s) =e= klem(t,r,s)*(uc_klem(t,r,s)/pkle(t,r,s))**se_klem(s) ;

* hicksian demand equation - materials(m) bundle
hde_mat(t,r,s)$(y0(r,s))..
   mat(t,r,s) =e= klem(t,r,s)*(uc_klem(t,r,s)/pmat(t,r,s))**se_klem(s) ;

* unit cost equation - capital(k)-labor(l)-energy(e)-materials(m) bundle
uce_klem(t,r,s)$(y0(r,s))..
   uc_klem(t,r,s) =e= (cs_klem(t,r,"mat",s)*pmat(t,r,s)**(1-se_klem(s))
                    +   cs_klem(t,r,"kle",s)*pkle(t,r,s)**(1-se_klem(s)))
                    **(1/(1-se_klem(s))) ;

* zero profit - capital(k)-labor(l)-energy(e)-materials(m) bundle
zp_klem(t,r,s)$(y0(r,s) and not (tfirst(t) and extant_share(r,s) eq 1))..
   uc_klem(t,r,s) =e= pklem(t,r,s);

* hicksian demand equation - capital(k)-labor(l) bundle
hde_kl(t,r,s)$(y0(r,s))..
   kl(t,r,s) =e= kle(t,r,s)*(uc_kle(t,r,s)/pkl(t,r,s))**se_kle(s);

* hicksian demand equation - primary energy(en)-electricity(e) bundle
hde_ene(t,r,s)$(y0(r,s))..
  ene(t,r,s) =e= kle(t,r,s)*(uc_kle(t,r,s)/pene(t,r,s))**se_kle(s);

* unit cost equation - capital(k)-labor(l)-energy(e) bundle
uce_kle(t,r,s)$(y0(r,s))..
  uc_kle(t,r,s) =e= (cs_kle(t,r,"ene",s)*pene(t,r,s)**(1-se_kle(s))
                 +   cs_kle(t,r,"kl",s) *pkl(t,r,s) **(1-se_kle(s)))
                 **(1/(1-se_kle(s))) ;

* zero profit - capital(k)-labor(l)-energy(e) bundle
zp_kle(t,r,s)$(y0(r,s) and not (tfirst(t) and extant_share(r,s) eq 1))..
  uc_kle(t,r,s) =e= pkle(t,r,s) ;

* hicksian demand equation - labor for use with new capital
```

```
hde_ld(t,r,s)$(y0(r,s))..
  ld(t,r,s) =e= kl(t,r,s)*prod_ind(t,r,"l",s,"new")**(1-se_kl(s))
             * (uc_kl(t,r,s)*(1+tp0(r))/((1+tp(t,r))*pl(t,r)))**se_kl(s) ;

* hicksian demand equation - new capital
hde_kd(t,r,s)$(y0(r,s))..
  kd(t,r,s) =e= kl(t,r,s)*prod_ind(t,r,"k",s,"new")**(1-se_kl(s))
             * (uc_kl(t,r,s)*(1+tk0(r))*pr0/((1+tk(t,r))*pr(t,r)))**se_kl(s) ;

* unit cost equation - capital(k)-labor(l) bundle
uce_kl(t,r,s)$(y0(r,s) and not (tfirst(t) and extant_share(r,s) eq 1))..
  uc_kl(t,r,s) =e=
      (cs_kl(t,r,"l",s)*(prod_ind(t,r,"l",s,"new")*(1+tp(t,r))
   *pl(t,r)/(1+tp0(r)))**(1-se_kl(s))
   +   cs_kl(t,r,"k",s)*(prod_ind(t,r,"k",s,"new")*(1+tk(t,r))
   *pr(t,r)/((1+tk0(r))*pr0))**(1-se_kl(s)))**(1/(1-se_kl(s))) ;

* zero profit - capital(k)-labor(l) bundle
zp_kl(t,r,s)$(y0(r,s) and not (tfirst(t) and extant_share(r,s) eq 1))..
   uc_kl(t,r,s) =e= pkl(t,r,s) ;

* hicksian demand equation - materials intermediates
hde_id_m(t,r,sm,s)$(y0(r,s))..
   id(t,r,sm,s) =e= prod_ind(t,r,sm,s,"new")*mat(t,r,s) ;

* unit cost equation - materials(m) bundle
uce_mat(t,r,s)$(y0(r,s))..
  uc_mat(t,r,s) =e= sum(sm, cs_mat(t,r,sm,s)*prod_ind(t,r,sm,s,"new")*pa(t,r,sm)) ;

* zero profit - materials(m) bundle
zp_mat(t,r,s)$(y0(r,s) and not (tfirst(t) and extant_share(r,s) eq 1))..
   uc_mat(t,r,s) =e= pmat(t,r,s);

* hicksian demand equation - primary energy(en) bundle
hde_en(t,r,s)$(y0(r,s))..
   en(t,r,s) =e= ene(t,r,s)*(uc_ene(t,r,s)/pen(t,r,s))**se_ene(s);


* hicksian demand equation - electricity intermediates
hde_id_ele(t,r,sel,s)$(y0(r,s))..
   id(t,r,sel,s) =e= ene(t,r,s)*prod_ind(t,r,sel,s,"new")**(1-se_ene(s))
                  *  (uc_ene(t,r,s)/pa(t,r,sel))**se_ene(s) ;

* unit cost equation - primary energy(en)-electricity(e) bundle
uce_ene(t,r,s)$(y0(r,s))..
  uc_ene(t,r,s)
   =e= (cs_ene(t,r,"ele",s)*(sum(ss$sel(ss), prod_ind(t,r,ss,s,"new")
    *    pa(t,r,ss)))**(1-se_ene(s))
    +   cs_ene(t,r,"en",s)*pen(t,r,s)**(1-se_ene(s)))**(1/(1-se_ene(s))) ;

* zero profit - primary energy(en)-electricity(e) bundle
zp_ene(t,r,s)$(y0(r,s) and not (tfirst(t) and extant_share(r,s) eq 1))..
  uc_ene(t,r,s) =e= pene(t,r,s) ;

* hicksian demand equation - primary energy
hde_id_en(t,r,sen,s)$(y0(r,s))..
  id(t,r,sen,s) =e= en(t,r,s)*prod_ind(t,r,sen,s,"new")**(1-se_en(s))
                 *   (uc_en(t,r,s)/pa(t,r,sen))**se_en(s);

* unit cost equation - primary energy(en) bundle
uce_en(t,r,s)$(y0(r,s))..
  uc_en(t,r,s) =e=
      sum(sen, cs_en(t,r,sen,s)*(prod_ind(t,r,sen,s,"new")
```

```
   *     pa(t,r,sen))**(1-se_en(s)))**(1/(1-se_en(s))) ;

* zero profit - primary energy(en) bundle
zp_en(t,r,s)$(y0(r,s) and not (tfirst(t) and extant_share(r,s) eq 1))..
   uc_en(t,r,s) =e= pen(t,r,s);

*  ---------------------------------------------------------------------------
*
*     Abatement module -- all vintages
*
*  ---------------------------------------------------------------------------

* hicksian demand equation - intermediates for abatement
hde_id_abate(t,r,ss,s,v)$(id_abate0(t,r,ss,s,v))..
   id_abate(t,r,ss,s,v) =e= abate(t,r,s,v) ;

* hicksian demand equation - labor for abatement
hde_ld_abate(t,r,s,v)$(ld_abate0(t,r,s,v))..
   ld_abate(t,r,s,v) =e= abate(t,r,s,v) ;

* hicksian demand equation - capital for abatement
hde_kd_abate(t,r,s,v)$(kd_abate0(t,r,s,v))..
   kd_abate(t,r,s,v) =e= abate(t,r,s,v) ;

* unit cost equation - abatement
uce_abate(t,r,s,v)$(abate0(t,r,s,v))..
  uc_abate(t,r,s,v) =e=
      sum(ss, cs_abate(t,r,ss,s,v)*pa(t,r,ss))
  +   cs_abate(t,r,"l",s,v)*(1+tp(t,r))*pl(t,r)/(1+tp0(r))
  +   cs_abate(t,r,"k",s,v)*(1+tk(t,r))*pr(t,r)/((1+tk0(r))*pr0) ;

* zero profit - abatement
zp_abate(t,r,s,v)$(abate0(t,r,s,v))..
   uc_abate(t,r,s,v) =e= pabate(t,r,s,v) ;

*  ---------------------------------------------------------------------------
*
*     Make module
*
*  ---------------------------------------------------------------------------

*  Aggregate output
*  ytot(t,r,s) = y_ex(t,r,s)+y(t,r,s) ;

*  ---------------------------------------------------------------------------
*
*     Income module
*
*  ---------------------------------------------------------------------------

* budget constraint - household
* the handling of the capital endowment using the conditionals based on the
* length of the t set is to correctly specify the budget constraint for the
* static model where the value of the endowment is defined by pr and not pk
* for the dynamic model need to account for the endogenous value of the initial
* period new capital stock in sensitivity analyses where the capital stock in
* the first period is not all extant

variables
   phikdist(t,r,h)      "National distribution of 'new' profits"
   savings(t,r,h)       "Savings equation"
;
```

```
equations
   phikdisteq(t,r,h)    "National distribution of 'new' profits"
   saveq(t,r,h)         "Savings equation"
;

phikdisteq(t,r,h)$(not tfirst(t) and card(t) gt 1)..
   phikdist(t,r,h) =e= kh(t,r,h)*ke0(r,h)/sum((rr,hh), kh(t,rr,hh)*ke0(rr,hh)) ;

saveq(t,r,h)..
   0 =e= (kh(t+1,r,h)*ke0(r,h)  - (kh(t,r,h)*ke0(r,h) + savings(t,r,h)
      -     sum(rr, pr(t,rr)*k(t,rr)*k0(rr))*phikdist(t,r,h)))
      $(card(t) ne 1 and not tlast(t))
      +  (pkt(r)*kt(r)*ke0(r,h) - (kh(t,r,h)*ke0(r,h) + savings(t,r,h)
      -     sum(rr, pr(t,rr)*k(t,rr)*k0(rr))*phikdist(t,r,h)))
      $(card(t) ne 1 and tlast(t))
      +  (savings(t,r,h) - pkt(r)*invh0(r,h))
      $(card(t) eq 1)
      ;

bc_hh(t,r,h)..
   pcl(t,r,h)*cl(t,r,h)*cl0(r,h) + savings(t,r,h) =e=
*     Profits in comparative static model
      pr(t,r)*ke0(r,h)$(card(t) eq 1)
*     Profits from 'new' capital in dynamic model
  +   sum(rr, pr(t,rr)*k(t,rr)*k0(rr))*phikdist(t,r,h)
*     Profit from 'extant' capital
  +   pr_ex_agg(t,r)*k_ex(t,r)*ke0(r,h)
*     After tax labor income
  +   (1-tl(t,r,h)-tfica(t,r,h))*pl(t,r)*te(t,r,h)
*     Natural resource income
  +   sum(s, pres(t,r,s)*(res_ex(t,r,s)+res(t,r,s))*rese0(r,s,h))
*     Net balance of payment residual
  +   pfx(t)*bopdef(r,h)*q_base(t)
*     Government transfers
  +   cpi(t)*tran0(r,h)*q_base(t)
*     Adjustments to gross labor tax
  +   pl(t,r)*tl_refund(r,h)*q_base(t)
*     Lump sum taxes to close government accounts
  +   cpi(t)*incadj(t)*incadj_share(r,h)
*     Exogenous component
  +   inc_extra(t,r,h) ;

$ontext
bc_hh(t,r,h)..
  kh(t+1,r,h)*ke0(r,h)
  +   pkt(r)*kt(r)*ke0(r,h)$(tlast(t) and card(t) gt 1)
  +   pkt(r)*invh0(r,h)$(card(t) eq 1)
  +   pcl(t,r,h)*cl(t,r,h)*cl0(r,h)
 =e=
      kh(t,r,h)*ke0(r,h)$(ord(t) gt 1)
  +   pk(t,r)*kh(t,r,h)*ke0(r,h)$(ord(t) eq 1 and card(t) gt 1)
  +   pr(t,r)*ke0(r,h)$(card(t) eq 1)
  +   pr_ex_agg(t,r)*k_ex(t,r)*ke0(r,h)
  +   (1-tl(t,r,h)-tfica(t,r,h))*pl(t,r)*te(t,r,h)
  +   sum(s, pres(t,r,s)*(res_ex(t,r,s)+res(t,r,s))*rese0(r,s,h))
  +   pfx(t)*bopdef(r,h)*q_base(t)
  +   cpi(t)*tran0(r,h)*q_base(t)
  +   pl(t,r)*tl_refund(r,h)*q_base(t)
  +   cpi(t)*incadj(t)*incadj_share(r,h)
  +   inc_extra(t,r,h) ;
$offtext

* first order condition - consumption(c)-leisure(l) bundle
```

```
foc_cl(t,r,h)..
   (cl(t,r,h)*cl0(r,h)/n(t,r,h))**(-eta(r,h)) =e= lambda(t,r,h)*pcl(t,r,h) ;

* first order condition - savings
foc_lambda(t+1,r,h)..
   beta(t+1,r,h)*lambda(t+1,r,h) =e= beta(t,r,h)*lambda(t,r,h) ;

* market clearance - time
mc_time(t,r,h)..
   te(t,r,h) =e= leis(t,r,h)*leis0(r,h)+l(t,r,h)*l0(r,h) ;

*  ---------------------------------------------------------------------------
*
*     Demand module
*
*  ---------------------------------------------------------------------------

* hicksian demand equation - leisure
hde_leis(t,r,h)..
   leis(t,r,h) =e= cl(t,r,h)*(uc_cl(t,r,h)*(1-tl0(r,h)-tfica0(r,h))
                / ((1-tl(t,r,h)-tfica(t,r,h))*pl(t,r)))**se_cl(r,h);

* hicksian demand equation - consumption bundle
hde_c(t,r,h)..
   c(t,r,h) =e= cl(t,r,h)*(uc_cl(t,r,h)/pc(t,r,h))**se_cl(r,h);

* unit cost equation - consumption(c)-leisure(l) bundle
uce_cl(t,r,h)..
   uc_cl(t,r,h) =e= (cs_cl(r,h,"leis")*((1-tl(t,r,h)-tfica(t,r,h))*pl(t,r)
                 /   (1-tl0(r,h)-tfica0(r,h)))**(1-se_cl(r,h))
                 +   cs_cl(r,h,"c")*pc(t,r,h)**(1-se_cl(r,h)))
                 **(1/(1-se_cl(r,h)));

* zero profit - consumption-leisure bundle
zp_cl(t,r,h)$(cl0(r,h))..
   uc_cl(t,r,h) =e= pcl(t,r,h) ;

* hicksian demand equation - consumption materials bundle
hde_cm(t,r,h)..
   cm(t,r,h) =e= c(t,r,h)*(uc_c(t,r,h)/pcm(t,r,h))**se_c;

* hicksian demand equation - consumption electricity and energy bundle
hde_cene(t,r,h)..
   cene(t,r,h) =e= c(t,r,h)*(uc_c(t,r,h)/pcene(t,r,h))**se_c;

* unit cost equation - consumption bundle
uce_c(t,r,h)..
   uc_c(t,r,h) =e= (cs_c(t,r,h,"cm")*pcm(t,r,h)**(1-se_c)
                +   cs_c(t,r,h,"cene")*pcene(t,r,h)**(1-se_c))**(1/(1-se_c)) ;

* zero profit - consumption bundle
zp_c(t,r,h)$(c0(r,h))..
   uc_c(t,r,h) =e= pc(t,r,h) ;

* hicksian demand equation - consumption electricity
hde_cd_ele(t,r,sel,h)..
   cd(t,r,sel,h) =e= cene(t,r,h)*pref_ind(t,r,sel,h)**(1-se_cene)
                  * (uc_cene(t,r,h)*(1+tc0(r))/((1+tc(t,r))
                  *  pa(t,r,sel)))**se_cene;

* hicksian demand equation - consumption primary energy bundle
hde_cen(t,r,h)..
   cen(t,r,h) =e= cene(t,r,h)*(uc_cene(t,r,h)/pcen(t,r,h))**se_cene;
```

```
* unit cost equation - consumption electricity and energy bundle
uce_cene(t,r,h)..
  uc_cene(t,r,h) =e= (cs_cene(t,r,h,"ele")*((1+tc(t,r))
                  *  sum(s$sel(s), pref_ind(t,r,"ele",h)
                  *     pa(t,r,s)/(1+tc0(r))))**(1-se_cene)
                  +  cs_cene(t,r,h,"cen")
                  *     pcen(t,r,h)**(1-se_cene))**(1/(1-se_cene));

* zero profit - consumption electricity and energy bundle
zp_cene(t,r,h)$(cene0(t,r,h))..
   uc_cene(t,r,h) =e= pcene(t,r,h) ;

* hicksian demand equation - consumption energy
hde_cd_en(t,r,scen,h)..
   cd(t,r,scen,h) =e= cen(t,r,h)*pref_ind(t,r,scen,h)**(1-se_cen)
                   * (uc_cen(t,r,h)*(1+tc0(r))/((1+tc(t,r))
                   *    pa(t,r,scen)))**se_cen ;

* unit cost equation - consumption primary energy bundle
uce_cen(t,r,h)..
  uc_cen(t,r,h) =e= sum(scen, cs_cen(t,r,h,scen)
                 *   (pref_ind(t,r,scen,h)*(1+tc(t,r))*pa(t,r,scen)/(1+tc0(r)))
                 **     (1-se_cen))**(1/(1-se_cen));

* zero profit - consumption primary energy bundle
zp_cen(t,r,h)$(cen0(t,r,h))..
   uc_cen(t,r,h) =e= pcen(t,r,h) ;

* hicksian demand equation - consumption materials
hde_cd_m(t,r,scm,h)..
  cd(t,r,scm,h) =e= cm(t,r,h)*pref_ind(t,r,scm,h)**(1-se_cm)
                 *  (uc_cm(t,r,h)*(1+tc0(r))/((1+tc(t,r))*pa(t,r,scm)))**se_cm ;

* unit cost equation - consumption materials bundle
uce_cm(t,r,h)..
   uc_cm(t,r,h) =e=
      sum(scm, cs_cm(t,r,h,scm)*(pref_ind(t,r,scm,h)
         *  (1+tc(t,r))*pa(t,r,scm)/(1+tc0(r)))**(1-se_cm))**(1/(1-se_cm));

* zero profit - consumption materials bundle
zp_cm(t,r,h)$(cm0(t,r,h))..
   uc_cm(t,r,h) =e= pcm(t,r,h);

* hicksian demand equation - government
hde_g(t,r,s)..
   g(t,r,s) =e= gov(t,r) ;

* unit cost equation - government bundle
uce_gov(t,r)..
   uc_gov(t,r) =e= sum(s, cs_gov(t,r,s)*pa(t,r,s));

* zero profit - government bundle
zp_gov(t,r)$(gov0(r))..
   uc_gov(t,r) =e= pgov(t,r);

* hicksian demand equation - investment
hde_i(t,r,s)..
   i(t,r,s) =e= inv(t,r);

* unit cost equation - investment bundle
uce_inv(t,r)..
   uc_inv(t,r) =e= sum(s, cs_inv(r,s)*pa(t,r,s));
```

```
* zero profit - investment
zp_inv(t,r)$(inv0(r))..
   uc_inv(t,r) =e= pk(t+1,r) + pkt(r)$tlast(t);

* zero profit - capital stock
zp_k(t,r)$(card(t) gt 1 and not tfirst(t))..
  pk(t,r) =e= pr(t,r) + (pk(t+1,r) + pkt(r)$tlast(t))*(1-delta);

*  ---------------------------------------------------------------------------
*
*     Armington/CET module
*
*  ---------------------------------------------------------------------------

* market clearance - goods market
mc_a(t,r,s)$(a0(r,s))..
  a(t,r,s)*a0(r,s) =e= sum(ss, id(t,r,s,ss)*id_base(t,r,s,ss))
                     +  sum(ss, id_ex(t,r,s,ss)*id0(r,s,ss))
                     +  sum((ss,v), id_abate(t,r,s,ss,v)*id_abate0(t,r,s,ss,v))
                     +  sum(h, cd(t,r,s,h)*cd_base(t,r,s,h))
                     +  i(t,r,s)*i0(r,s) + g(t,r,s)*g_base(t,r,s);

* hicksian demand equation - imports
hde_m(t,r,s,trd)$(m0(r,s,trd))..
  m(t,r,s,trd) =e= (dn(t,r,s)*(uc_dn(t,r,s)/pn(t,s))**se_dn(s))$(dtrd(trd))
                 +  (a(t,r,s)*(uc_a(t,r,s)/(pfx(t)*pm(t,s)))**se_nf(s))
                 $(ftrd(trd)) ;

* hicksian demand equation - domestic-national armington aggregate
hde_dn(t,r,s)..
   dn(t,r,s) =e= a(t,r,s)*(uc_a(t,r,s)/pdn(t,r,s))**se_nf(s);

* unit cost equation - armington aggregate
uce_a(t,r,s)..
   uc_a(t,r,s) =e= (cs_nf(r,"n",s)*pdn(t,r,s)**(1-se_nf(s))
                 +   cs_nf(r,"f",s)*(pfx(t)*pm(t,s))**(1-se_nf(s)))
                 **(1/(1-se_nf(s))) ;

* zero profit - armington aggregate
zp_a(t,r,s)$(a0(r,s))..
   uc_a(t,r,s) =e= pa(t,r,s) ;

* unit cost equation - domestic-national armington aggregate
uce_dn(t,r,s)..
   uc_dn(t,r,s) =e= ((cs_dn(r,"d",s)*pd(t,r,s)**(1-se_dn(s)))$(cs_dn(r,"d",s))
                  +   (cs_dn(r,"n",s)*pn(t,s)**(1-se_dn(s)))$(cs_dn(r,"n",s)))
                  **  (1/(1-se_dn(s))) ;

* zero profit - domestic-national armington aggregate
zp_dn(t,r,s)$(dn0(r,s))..
  uc_dn(t,r,s) =e= pdn(t,r,s);

* hicksian demand equation - domestic supply
hde_d(t,r,s)$(d0(r,s))..
  d(t,r,s) =e= dn(t,r,s)*(uc_dn(t,r,s)/pd(t,r,s))**se_dn(s);

* market clearance - domestic output
mc_d(t,r,s)$(y0(r,s))..
   d(t,r,s) =e= (y_ex(t,r,s)+y(t,r,s))*(pd(t,r,s)/py(t,r,s))**te_dx(s) ;

* hicksian demand equation - exports
hde_x(t,r,s,trd)$(x0(r,s,trd))..
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   x(t,r,s,trd) =e= (y_ex(t,r,s)+y(t,r,s))
                  *  (((pn(t,s)/py(t,r,s))**te_dx(s))$(dtrd(trd))
                  +   ((pfx(t)*px(t,s)/py(t,r,s))**te_dx(s))$(ftrd(trd)));

* unit cost equation - destination differentiated output cet
uce_dx(t,r,s)$(y0(r,s))..
  py(t,r,s) =e= (cs_dx(r,"d",s)*pd(t,r,s)**(1+te_dx(s))
             +   cs_dx(r,"n",s)*pn(t,s)         **(1+te_dx(s))
             +   cs_dx(r,"f",s)*(pfx(t)*px(t,s))**(1+te_dx(s)))
             **(1/(1+te_dx(s))) ;

* market clearance - national trade
mc_dtrd(t,s)..
   sum((r,dtrd), x(t,r,s,dtrd)*x0(r,s,dtrd))
      =e= sum((r,dtrd), m(t,r,s,dtrd)*m0(r,s,dtrd)) ;

*  ------------------------------------------------------------------------------
*
*     Factor markets
*
*  ------------------------------------------------------------------------------

* market clearance - labor
mc_l(t,r)..
  sum(h, l(t,r,h)*l0(r,h)) =e=  sum(s, ld_ex(t,r,s)*ld0(r,s))
                            +   sum(s, ld(t,r,s)*ld_base(t,r,s))
                            +   sum((s,v), ld_abate(t,r,s,v)*ld_abate0(t,r,s,v)) ;

*  Capital

* market clearance - rental rate for sector differentiated extant capital
mc_pr_ex(t,r,s)$(extant_share(r,s) and y0(r,s))..
  kd_ex(t,r,s) =e= k_ex(t,r)*extant_share(r,s)/extant_share_agg(r)
                * (pr_ex(t,r,s)/pr_ex_agg(t,r))**te_k_ex ;

* unit cost equation - sector differentiated extant capital
uce_k_ex(t,r)$(extant_share_agg(r))..
  pr_ex_agg(t,r)/pr0 =e= sum(s, cs_kd_ex(r,s)*(pr_ex(t,r,s)/pr0)**(1+te_k_ex))
                      **   (1/(1+te_k_ex)) ;


* market clearance - rental rate for new capital
mc_pr(t,r)..
  k(t,r)*k0(r) =e= sum(s, kd(t,r,s)*kd_base(t,r,s))
                +  sum((s,v), kd_abate(t,r,s,v)*kd_abate0(t,r,s,v)) ;

* market clearance - price of new capital
mc_pk(t,r)..
  k(t,r)*k0(r) =e= k(t-1,r)*k0(r)*(1-delta) + inv(t-1,r)*inv0(r)
                +  ((1-extant_share_agg(r))*k0(r))$tfirst(t) ;

* market clearance - price of post-terminal capital
mc_pkt(r)..
  kt(r)*k0(r) =e= sum(t$tlast(t), k(t,r)*k0(r)*(1-delta) + inv(t,r)*inv0(r)) ;

*  Natural resources

* market clearnace - fixed factor resources
mc_pres(t,r,s)..
  res0(r,s) =g= (res_ex(t,r,s)+res(t,r,s))*res0(r,s) ;


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*  Model closure
* ------------------------------------------------------------------------------

* market clearnace - foreign trade
mc_fx(t)$(not tfirst(t))..
  sum((r,s,ftrd), px(t,s)*x(t,r,s,ftrd)*x0(r,s,ftrd))
    +  sum((r,h), bopdef(r,h)*q_base(t))
   =e= sum((r,s,ftrd), pm(t,s)*m(t,r,s,ftrd)*m0(r,s,ftrd)) ;

* budget constraint - government
bc_gov(t)..
  sum(r, pgov(t,r)*gov(t,r)*gov0(r))
    =e= sum(r, sum(s, tp(t,r)*pl(t,r)*ld(t,r,s)*ld_base(t,r,s)
     +                tp(t,r)*pl(t,r)*ld_ex(t,r,s)*ld0(r,s)
     +                tp(t,r)*pl(t,r)*sum(v, ld_abate(t,r,s,v)*ld_abate0(t,r,s,v))
     +                tk(t,r)*pres(t,r,s)*(res_ex(t,r,s)+res(t,r,s))*res0(r,s)
     +                tk(t,r)*pr(t,r)*kd(t,r,s)*kd_base(t,r,s)
     +                tk(t,r)*pr_ex(t,r,s)*kd_ex(t,r,s)*kd0(r,s)
     +                tk(t,r)*pr(t,r)*sum(v, kd_abate(t,r,s,v)*kd_abate0(t,r,s,v))
     +                ty(t,r,s)*py(t,r,s)*y_ex(t,r,s)*y0(r,s)
     +                ty(t,r,s)*py(t,r,s)*y(t,r,s)*y0(r,s)
     +                sum(h, tc(t,r)*pa(t,r,s)*cd(t,r,s,h)*cd_base(t,r,s,h)))
     +                sum(h, tl(t,r,h)*pl(t,r)*l(t,r,h)*l0(r,h)
     +                       tfica(t,r,h)*pl(t,r)*l(t,r,h)*l0(r,h))
     -                sum(h, cpi(t)*tran0(r,h)*q_base(t)
     +                pl(t,r)*tl_refund(r,h)*q_base(t)))
     -                cpi(t)*incadj(t) ;

* closure rule - real government expenditures per capita are fixed
cr_gov(t,r)..
   gov(t,r)*gov0(r) =e= q_base(t)*gov0(r) + gov_extra(t,r);

* closure rule - terminal capital
* scale the level of the terminal capital stock to achieve steady-state growth
* in last period investment.
cr_kt(r)..
   sum(tlast(t), inv(t,r)*sum(h, c(t-1,r,h)))
      =e= sum(tlast(t), sum(h, c(t,r,h))*inv(t-1,r)) ;

* ------------------------------------------------------------------------------
* auxilary
* ------------------------------------------------------------------------------

* auxilary - extant capital
aux_k_ex(t,r)$(extant_share_agg(r))..
   k_ex(t,r) =e= extant_share_agg(r)*(1-delta)**(ord(t)-1);

* auxilary - welfare
aux_w(r,h)..
   w(r,h) =e= sum(t,  n(t,r,h)*beta(t,r,h)
           *   (cl(t,r,h)*cl0(r,h)/n(t,r,h))**(1-eta(r,h))/(1-eta(r,h))) ;

* auxilary - consumer price index
aux_cpi(t)..
  cpi(t) =e= sum((rr,hh), pc(t,rr,hh)*c0(rr,hh))/sum((rr,hh), c0(rr,hh)) ;
```