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Alternative Presentation of PM Expert Elicitation Results

Presentation to EPA Science Advisory
Board 812 Council Health Effects
Subcommittee

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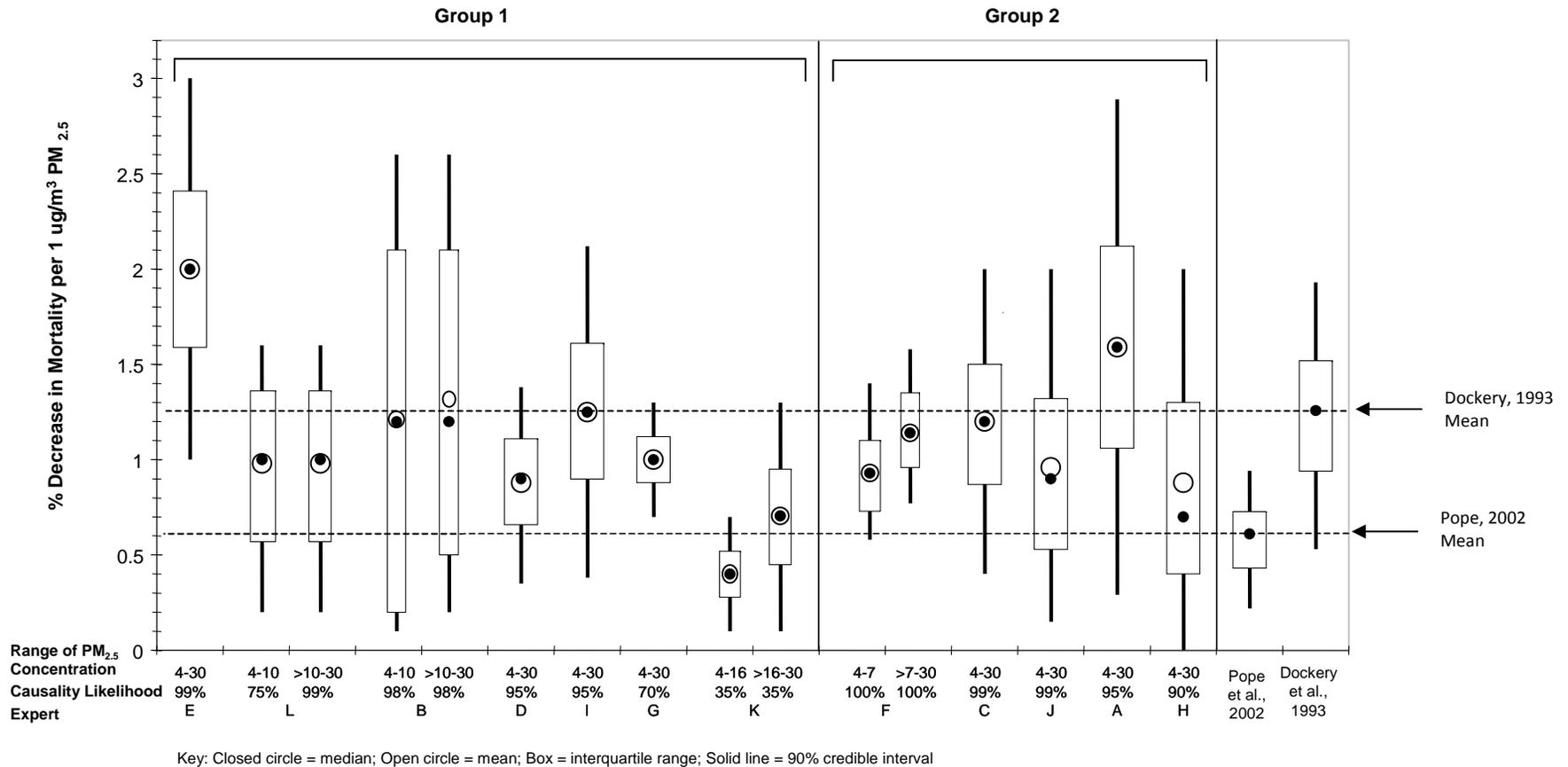
Background

- Major Expert Elicitation (EE) study of the mortality effects of PM_{2.5} completed by EPA in 2006.
- Elicited subjective probabilistic distributions of uncertainty in PM-mortality concentration response coefficient for use in EPA benefits analyses.
- 12 experts, 12 distributions (A - L)
- Individual expert distributions programmed in BenMAP, applied (unpooled) in subsequent analyses (PM, NAAQS RIA; RSM-based PM co-benefits in other RIAs).
- Lack of combined estimate poses presentation challenges
 - Reporting of 12 distributions can be cumbersome.
 - SAB critiqued EE range reported in PM NAAQS as misleading.

Past SAB Advice

- Excerpt from EPA SAB PM NAAQS RIA consultation in 2008:
 - “Where experts largely agree, it would be appropriate to collapse the various estimates into a single distribution (or point estimate with uncertainty bounds) while still providing the individual estimates elsewhere...In future analyses, the decision about aggregation must be made in the context of each analysis and its purpose.”
- Is aggregation a reasonable approach for the 812 analysis?
Is there a viable means of combining the PM EE results?

Results of the PM Expert Elicitation (2006)



Note: Box plots represent distributions as provided by the experts to the elicitation team. Experts in Group 1 preferred to give conditional distributions and keep their probabilistic judgment about the likelihood of a causal or non-causal relationship separate. Experts in Group 2 preferred to give distributions that incorporate their likelihood that the PM_{2.5} mortality association may be non-causal. Therefore, the expert distributions from these two groups are not directly comparable.

Challenges

- PM EE study not designed to yield “combinable” estimates
 - No test or “seed” questions in protocol
 - No self- or peer-weights
 - Consensus not an objective
 - Allowed for variation in:
 - Shape of C-R function
 - Threshold
 - Treatment of Causal Probability
- Likely significant dependence among expert responses.

Options for Combining Results

- Substantial literature from 80s onward (Genest and Zidek, Clemen and Winkler, Cooke, Jouini and Clemen) but little agreement on whether and how to combine distributions mathematically
- Choices
 - Linear opinion pool
 - Logarithmic opinion pool
 - Cooke's classical method
 - Copula functions

Opinion Pooling

- Linear opinion pool

$$f(\theta) = \sum_{i=1}^n w_i f_i(\theta)$$

- Weighted average of individual distributions using subjective weights (e.g., equal weighting)
- Useful where other weights are lacking
- Equal weights potentially appropriate for public policy analysis
- Can perform as well as more complex methods (Clemen, 1989)
- Does not account for dependence among experts (may overweight some views)
- Tends to broaden distributions

Opinion Pooling (cont'd)

- Logarithmic Opinion Pool

$$f(\theta) = k \prod_{i=1}^n f_i(\theta)^{w_i}$$

- Derives a combined distribution by taking a weighted geometric mean of a set of individual distributions
- Weights can be subjective, including equal weights
- Not designed to address dependence among experts
- “Single Expert Veto”: any values considered implausible by any one expert are zeroed out in the pooled distribution (O’Hagan et al., 2006)
- Tends to produce narrower distributions, projecting greater knowledge
- Rarely used

Other Approaches

- Cooke's method
 - Requires performance measures based on responses to seed questions
- Copula functions
 - First proposed by Jouini and Clemen (1996); Also Hammitt and Shlyakhter, 1999).
 - A copula is "a mathematical function that can be used to represent probabilistic dependence when coupling marginal probability distributions (the experts' judgments) into a multivariate distribution (the joint likelihood of the experts' judgments)." (Hammitt and Shlyakhter, 1999).
 - Flexible; does not restrict the form of the expert distributions
 - Incorporates dependence among experts
 - Can exhibit the single-expert veto

Example Application of Copula Function

- Many copula functions exist. We used same form as Hammitt and Shlyakhter and Jouini and Clemen:

$$f_n(\theta) = kC_{n|\alpha} [1 - H_1(\theta), 1 - H_2(\theta), \dots, 1 - H_n(\theta)]h_1(\theta)h_2(\theta) \dots h_n(\theta) \quad (1)$$

$$C_{n|\alpha}(u_1, u_2, \dots, u_n) = \log_{\alpha} \left[1 + \frac{(\alpha^{u_1} - 1) \dots (\alpha^{u_n} - 1)}{(\alpha - 1)^{n-1}} \right] \quad (2)$$

- Where:
 - $H_i(\theta)$ = expert i 's CDF, evaluated at θ
 - $h_i(\theta)$ = expert i 's PDF, evaluated at θ
 - α = measure of dependence (0 =complete dependence; 1 = complete independence)
 - n = number of experts
 - k = normalization constant
 - All experts treated as equally dependent or independent

Approach

1. Derive PDFs/CDFs for C-R coefficients. Obtain mathematical expression of $h_i(\theta)$ and $H_i(\theta)$ for each expert.
2. Input PDFs/CDFs into copula. Evaluate across range of thetas.
3. Normalize copula. Set k so area under curve = 1.
4. Make BenMAP compatible. Convert function for input into BenMAP.
5. Repeat for different baseline PM levels
 - $PM > 16 \mu\text{g}/\text{m}^3$
 - $10 < PM \leq 16 \mu\text{g}/\text{m}^3$
 - $7 < PM \leq 10 \mu\text{g}/\text{m}^3$
 - $PM \leq 7 \mu\text{g}/\text{m}^3$
6. Run BenMAP. Pool Copula results across baseline PM levels.

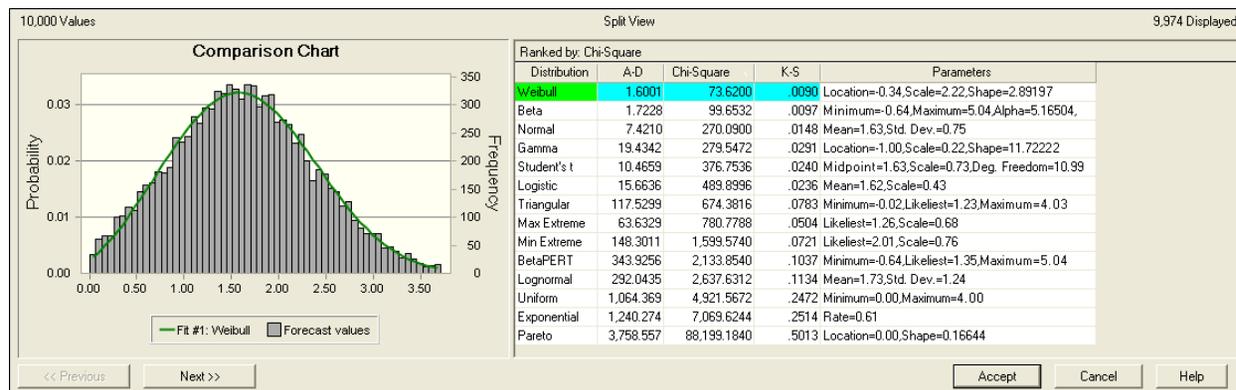
Derivation of PDFs and CDFs

- Challenges
 - Some experts provided fractiles (as requested) of an unspecified distributional form.
 - Even experts who specified parametric distributions modified them in some way.
 - Some are truncated.
 - About half the experts gave distributions conditional on a causal relationship.
 - One expert specified a probabilistic threshold.
- The Good News
 - Re-ran 812 CMAQ core scenario results through BenMAP with no threshold configuration for expert K. *Results differ only minimally from applying threshold.* Can reasonably assume no threshold for this application.

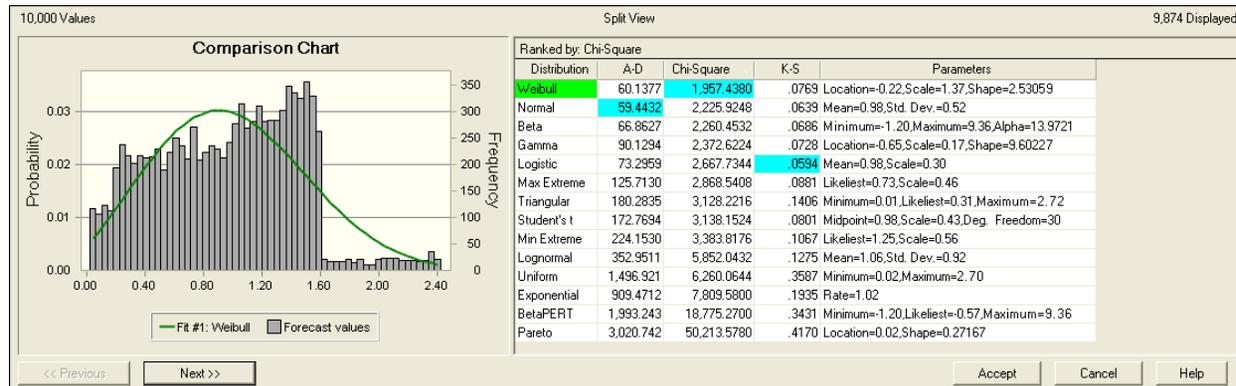
Derivation of PDFs and CDFs (cont'd)

- Used Crystal Ball™ to:
 - sample from elicited distributions (n = 10,000)
 - Fit distributions to sample output

A

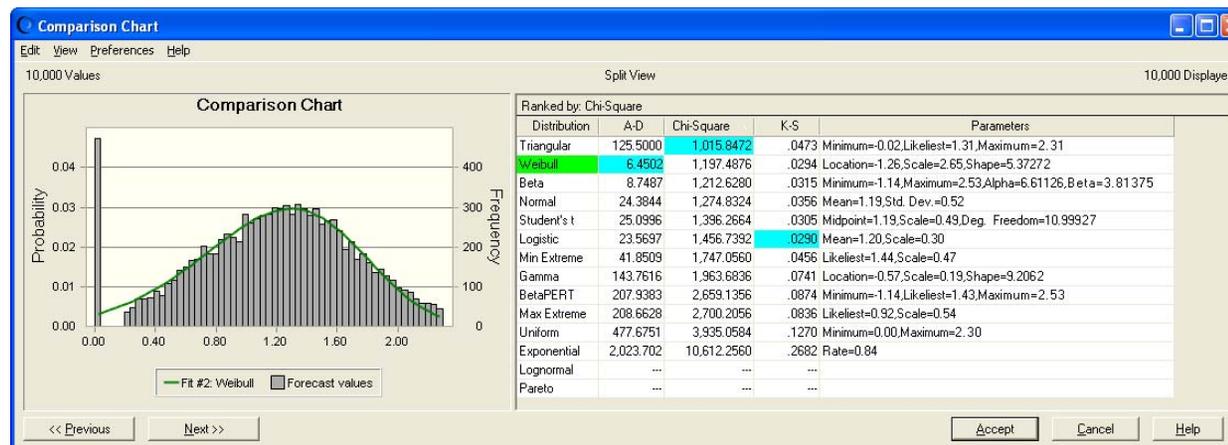


F
(high)



Causality

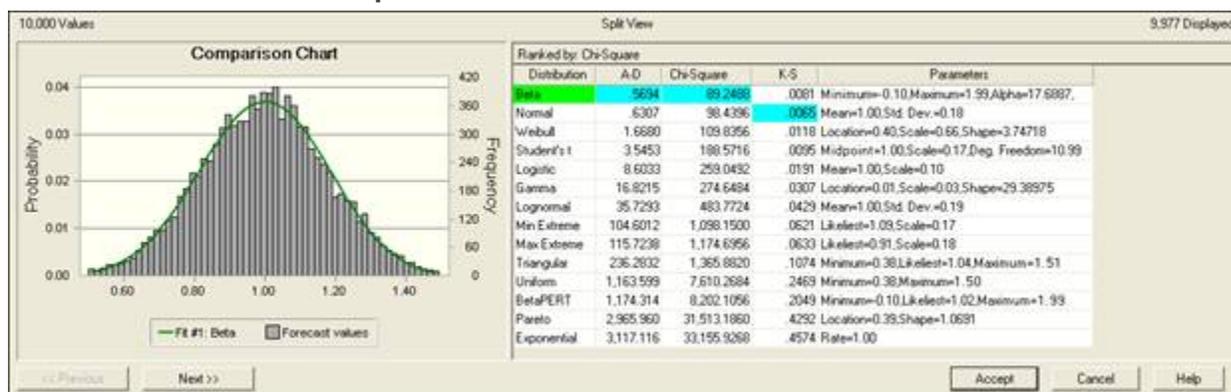
- For conditional distributions, we opted not to incorporate $p(\text{causal})$ before fitting.



- Instead, chose to fit conditional distributions and represent pdf as a combination of a discrete probability at zero and an adjusted pdf for positive values.

Causality Example

- Expert G(Conditional, $P(\text{causal}) = 0.7$); Fit Beta distribution to his conditional sample



- G PDF:
 - If $\theta = 0$, $h_g(\theta)$ based on narrow rectangular slice at zero, such that area = 0.3. Does not overlap rest of pdf.
 - For positive θ within the bounds of the Beta distribution, $h_g(\theta)$ equals 0.7 times the output of the Beta pdf at θ .
- G CDF:
 - If $\theta = 0$, $H_g(\theta) = 0.3$
 - For positive θ within the bounds of the Beta distribution, $H_g(\theta) = 0.3 + 0.7$ times the output of the Beta cdf at θ .

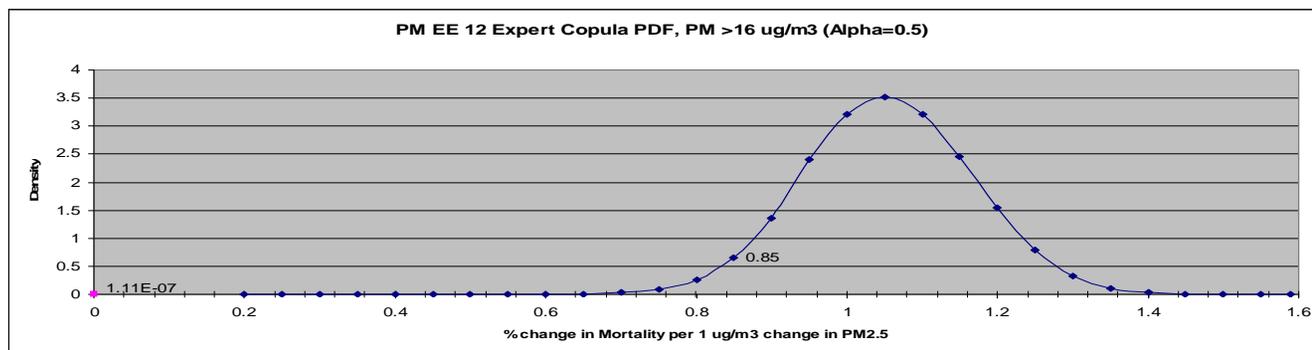
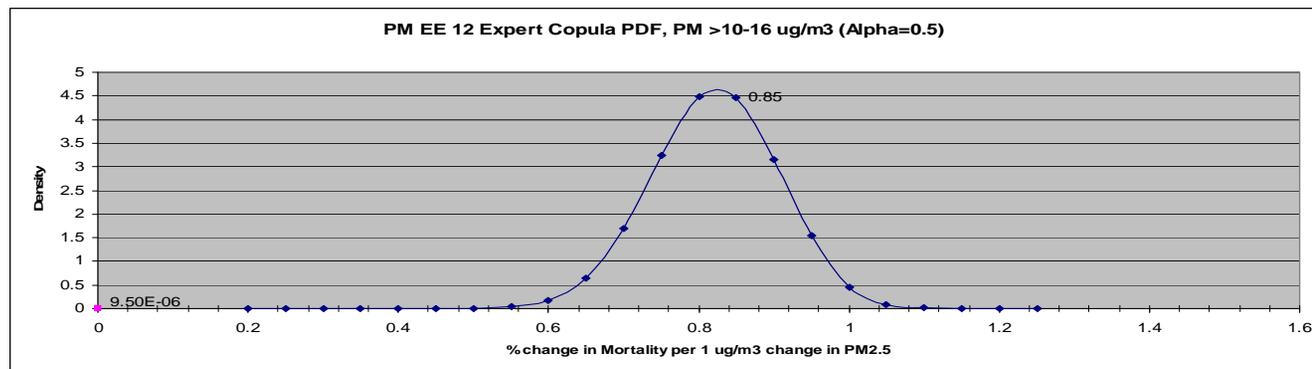
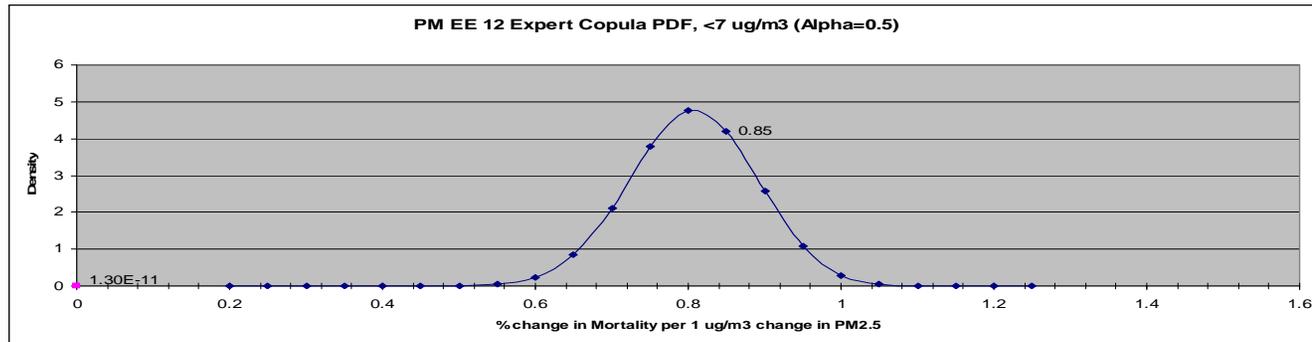
Fitted Expert Distributions

Expert	Distribution	Conditional?	$P(\theta = 0)$	PDF/CDF adjustment
A	Weibull	No	N/A	N/A
B(4-10)	Beta	Yes	0.02	0.98
B(>10-30)	Beta	Yes	0.02	0.98
C	Weibull	No	N/A	N/A
D	Triangular	Yes	0.05	0.95
E	Beta	Yes	0.01	0.99
F(>7-30)	Beta	No	N/A	N/A
F(<7)	Gamma	No	N/A	N/A
G	Beta	Yes	0.3	0.7
H	Beta	No	N/A	N/A
I	Beta	Yes	0.05	0.95
J	Beta	No	N/A	N/A
K(4-16)	Weibull	Yes	0.65	0.35
K(>16-30)	Weibull	Yes	0.65	0.35
L(4-10)	Beta	Yes	0.25	0.75
L(<10-30)	Weibull	Yes	0.01	0.99

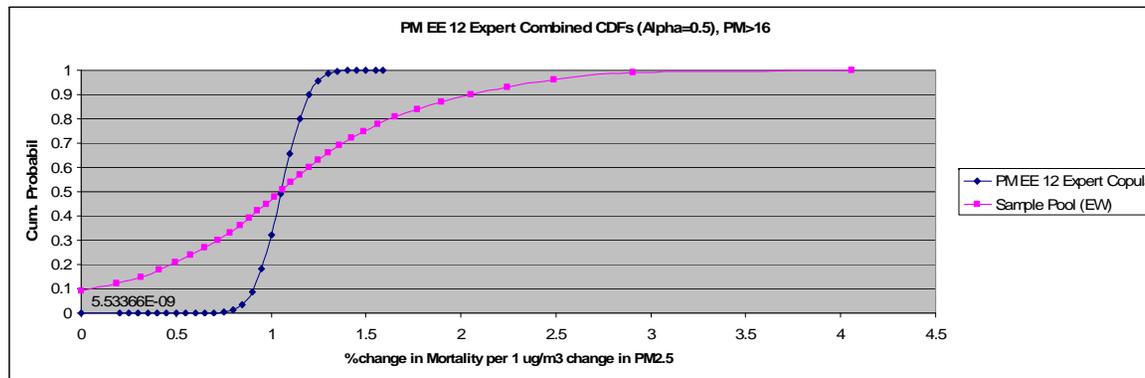
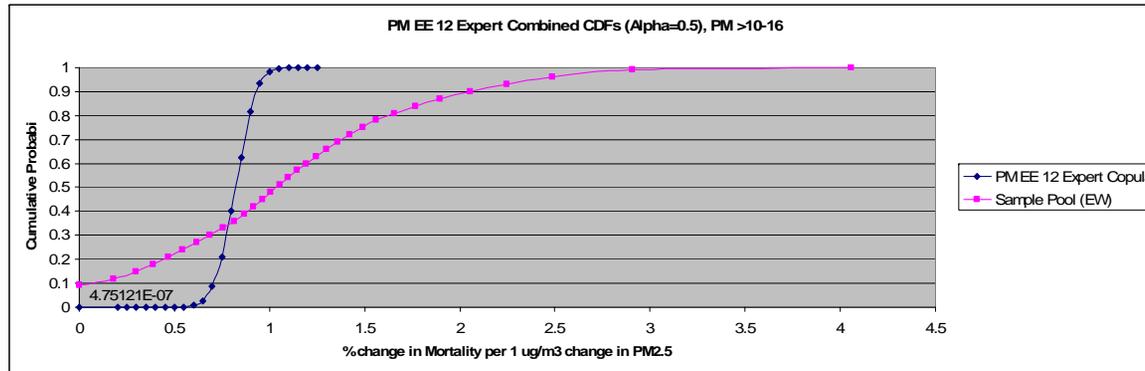
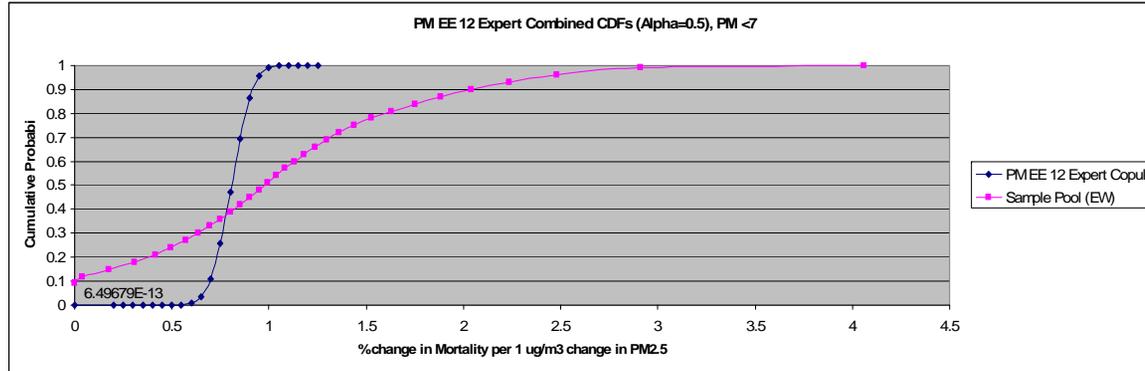
Calculating $F_n(\theta)$

- Developed spreadsheet model to calculate $h_i(\theta)$ and $H_i(\theta)$ for each expert and feed into non-normalized copula function $F(\theta)$.
- Identified θ that maximized $F(\theta)$ for a given α ; used to select range of θ s.
- Calculated $F(\theta)$ for uniformly spaced range of θ s.
- “Integrated” resulting curve using trapezoidal approximation and summing areas of each segment to get AUC.
- Normalized $F(\theta)$ by setting $k=1/\text{sum of non-normalized AUC}$.
- Calculated $F_n(\theta)$ for range of θ 's. Result is copula PDF.
- Estimated AUC for $F_n(\theta)$; plotted cumulative AUC for copula CDF.

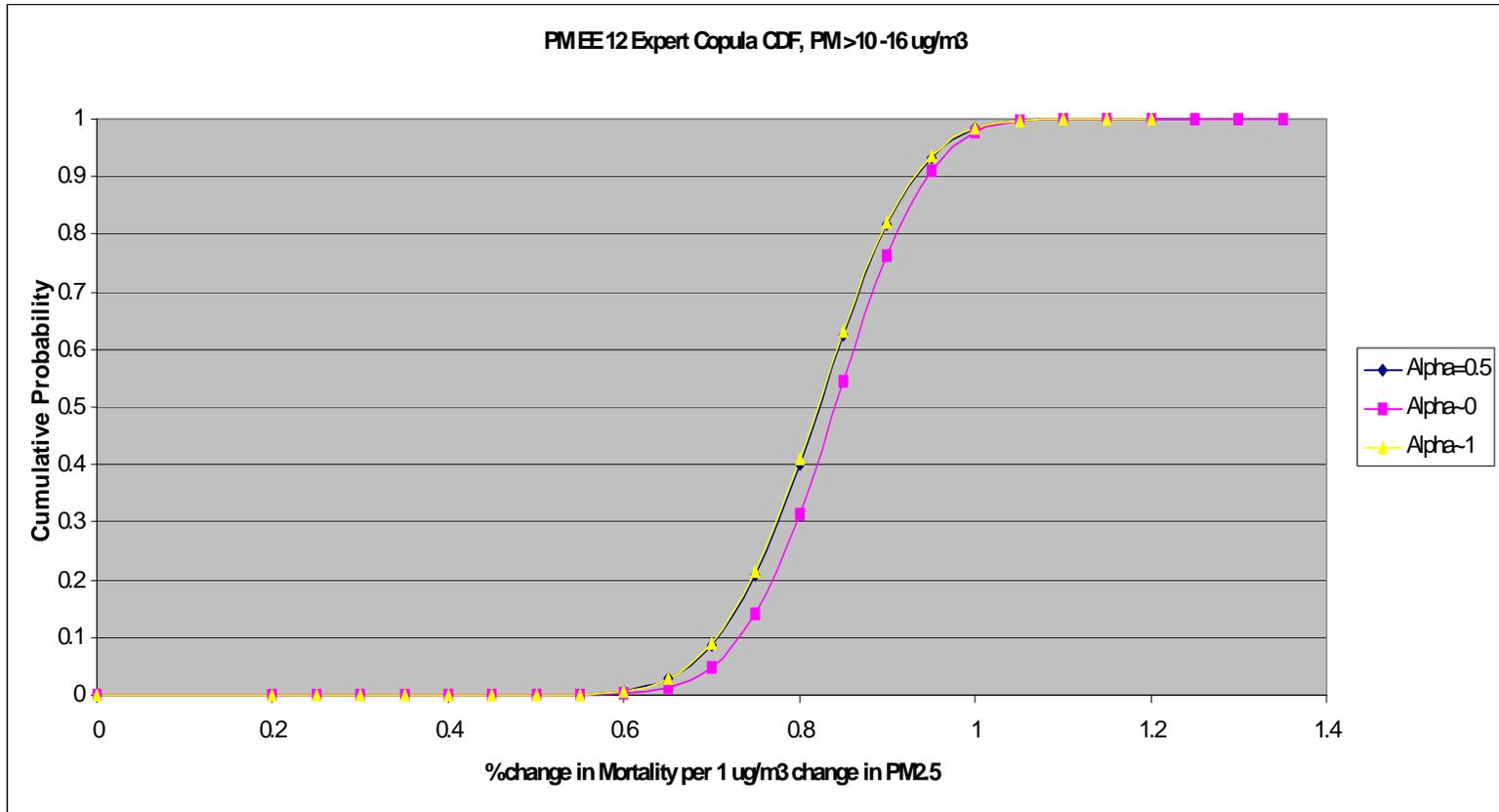
Copula Combined PDFs



Copula Combined CDFs Compared to Equal Weight (Linear) Pooling



Sensitivity Analysis (alpha)



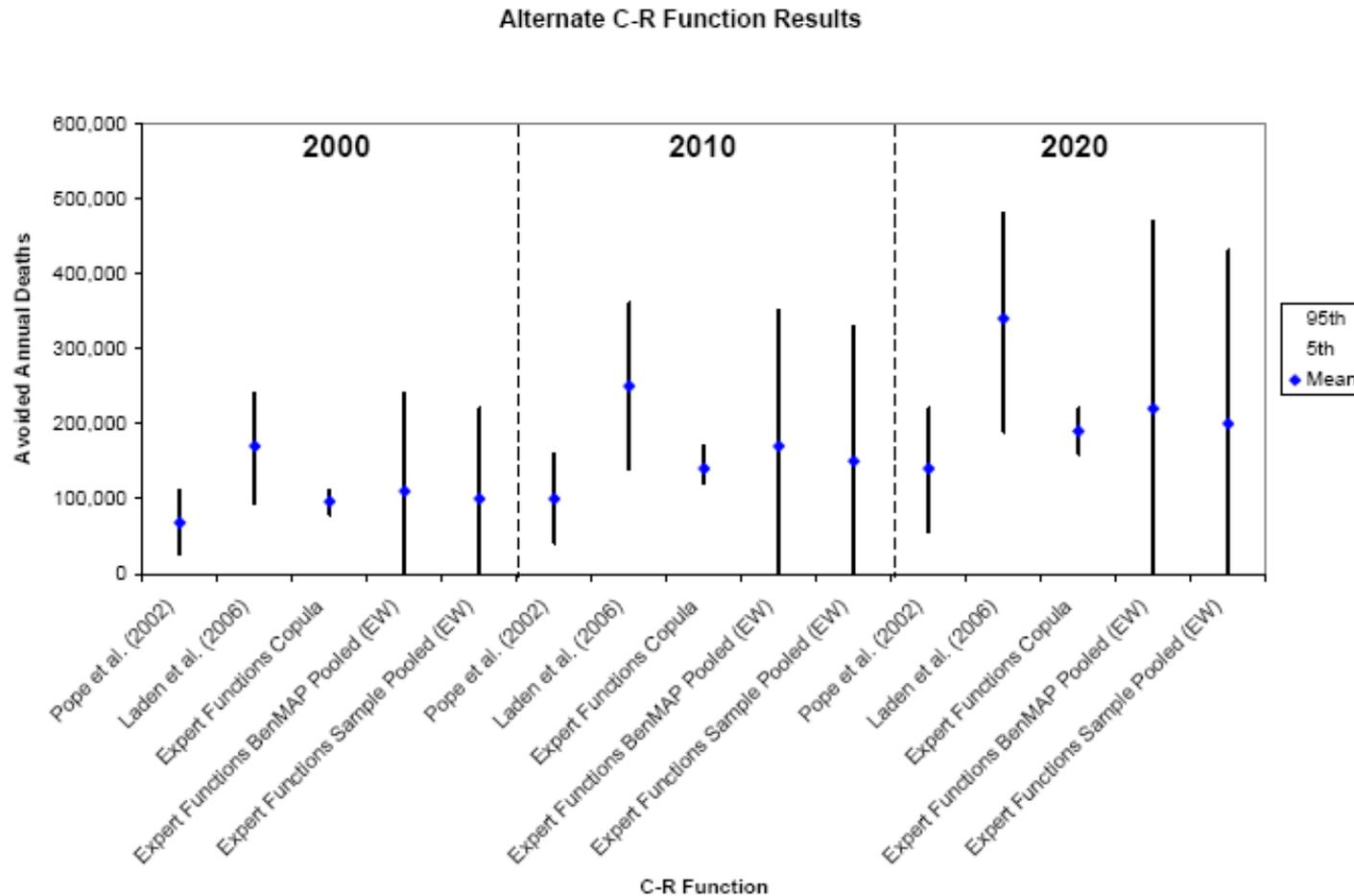
- Results do not appear sensitive to assumptions about dependence.

BenMAP Results

- Copula results for PM C-R coefficient were fed back through Crystal Ball™ to generate a percentile for input into BenMAP.
- Results were pooled across all PM levels in BenMAP.

C-R FUNCTION	2000			2010			2020		
	PERCENTILE 5	MEAN	PERCENTILE 95	PERCENTILE 5	MEAN	PERCENTILE 95	PERCENTILE 5	MEAN	PERCENTILE 95
Pope et al. (2002)	27,000	68,000	110,000	41,000	100,000	160,000	56,000	140,000	220,000
Laden et al. (2006)	94,000	170,000	240,000	140,000	250,000	360,000	190,000	340,000	480,000
Expert Functions Copula	79,000	96,000	110,000	120,000	140,000	170,000	160,000	190,000	220,000
Expert Functions BenMAP Pooled (EW)	0	110,000	240,000	0	170,000	350,000	0	220,000	470,000
Expert Functions Sample Pooled (EW)	0	100,000	220,000	0	150,000	330,000	0	200,000	430,000

Avoided Mortality Comparison



Summary

- Example Copula application produces central estimate of C-R coefficient consistent with PM EE study results.
- However, produces a dramatically narrower distribution. Different analytical choices may yield alternative results (e.g., alternative functional forms for the copula, adjustments to tails of distributions to account for potential overconfidence).
- Accounts for dependence, but results evaluated across all 12 experts insensitive to those assumptions. However, some subsets of experts may exhibit greater dependence than the group as a whole.
- Equal weighting dramatically broadens distribution, by comparison
- Seeking guidance on next steps

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