Exact Welfare Measures for a Policy Ban Revisited

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Abstract

Welfare measures for eliminating (or providing) a good based on observable data are discussed. The key is to determine the price range for integration by recovering the choke price of the Hicksian demand. The formulae to compute exact welfare measures, namely the compensating and equivalent variation, for banning a good based on commonly used single-equation demand models are provided. Unlike the Willig bounds of consumer surplus for a fixed price change, in the case of a policy ban, it is questionable whether consumer surplus can be a reasonable approximation for the exact welfare measures.

Keywords: Marshallian demand functions, elimination of a good, consumer surplus, compensating and equivalent variation, choke price, duality theory

JEL Codes: D60

1. Introduction

One of the fundamental questions to be answered when policy makers determine whether to eliminate (or grant) access to a good--private or public--is the benefits of the decision. This note is motivated by common practices of welfare analysis in some areas of demand research such as recreation demand analysis. One common practice is to estimate the Marshallian demand equation with a popular functional form (e.g., linear or semi-log), then use the consumer surplus (CS) to approximate Hicksian measures for welfare analysis of, for example, the elimination of a recreation site. In theory, the elimination of a good is to force the quantity consumed to be zero or, equivalently to raise its price so high that there is no consumption of the good. The minimum price necessary to eliminate consumption is often referred to as the choke price.

For a given price change, Willig (1976) shows that the CS, the area under the Marshallian demand curve between the current and new prices, provides a reasonable approximation for the exact welfare measures, compensating variation (CV) and equivalent variation (EV), when income effects are small. Hausman (1981) applies duality theory and derives CV and EV for a given price change from linear and log-linear Marshallian demand functions. Using the Hausman approach, Bockstael et al. (1985) tabulate the expenditure functions, indirect utility functions, along with the exact welfare measures for a given price change based on linear, semi-log, and log-linear Marshallian demand models.[1] In the case of banning a good, it is equivalent to raising its price from the current level to the choke price. What makes a policy ban different from a fixed price change is that the choke prices can differ for the Marshallian and Hicksian demands, resulting in different ranges of integration of

[1] Even though the exact welfare measures of a given price change based on these simple empirical specifications of Marshallian demand models are readily available, CS is still wildly used by practitioners for convenience. Subsequent theoretical studies of exact welfare measures focus mainly on the divergence of CV and EV (e.g.,

demands in computing CS and CV. The conventional Willig bounds (1976) of CS for a given price change do not directly apply in this case.

The purpose of this study is to examine the partial equilibrium welfare effects when a good is to be eliminated from (or provided for) consumption. Formulae to compute the exact welfare measures CV and EV for a policy ban based on observable market data are provided. This note extends the welfare analysis in Bockstael et al. (1984) for a given price change to the analysis of a policy ban, which requires raising the price along the Hicksian demand curve to its choke price that can be different from the choke price on the corresponding Marshallian demand curve. In this note, the exact welfare measures for a policy ban are derived for four commonly used demand models and compared with the associated CS measures. Consequently, unlike the Willig bounds of CS for a fixed price change, whether CS can be a reasonable approximation for CV and EV in the case of a policy ban is questionable.

2. Exact Welfare Measures for Elimination of a Good

The partial equilibrium welfare effect of eliminating a good is illustrated with a linear demand model in Figure 1. Let D^M and D^H be the Marshallian and corresponding Hicksian demand curves, respectively. The initial price of the good faced by an individual is P_0. Assume that the initial income level is I_0 and the initial consumption of the good is $X_0=X^M(P_0,I_0)=X^H(P_0,U_0)$. Let $U=U(P, I)$ and $I=I(P, U)$ be the indirect utility and expenditure functions, respectively. The initial level of utility is $U_0=U(P_0, I_0)$. Eliminating (banning) the good is equivalent to raising its price to the choke price P_M for the Marshallian demand that the CS is derived by integrating the area under the D^M curve from P_0 to P_M, resulting area *a*+*b* in Figure 1.

Hanemann (1991) and Shogren et al. (1994)).

Holding utility constant at the initial level, the CV is the area under the D^H curve from P_0 to P_H, resulting area *a*+*b*+*c* in Figure 1. As seen, the choke price P_H for the corresponding Hicksian demand at the initial utility level differs from P_M so that a different price range is integrated to derive CV.[2] The other exact welfare measure EV is the dollar amount necessary to compensate for the price increase to choke off consumption, holding the utility at the new level of no consumption of this good. Consequently EV is the area, *a*, in Figure 1. The two exact welfare measures can be computed as follows.

$$CV \quad = \quad \int_{P_0}^{P_H} \frac{\partial I(P,U_0)}{\partial P} dP \; = I(P_H, U_0) - I(P_0, U_0) \quad = \quad I(P_H, U(P_0,I_0)) - I_0 \qquad (1)$$

$$EV \quad = \quad \int_{P_0}^{P_M} \frac{\partial I(P,U_1)}{\partial P} dP \; = I(P_M, U_1) - I(P_0, U_1) \quad = \quad I_0 - I(P_0, U(P_M, I_0)) \qquad (2)$$

The above expression of CV and EV ensures positive welfare measures for a price increase.[3] The choke prices differ for the computation of CS and CV, while the choke price is the same between CS and EV because the new reference utility level is determined by $P=P_M$. It is seen in Figure 1 that EV≤CS≤CV. In this case, CV is an individual's willingness to accept (WTA) for the elimination and EV is the willingness to pay (WTP) for avoiding the elimination of the good.[4]

The observable market data enable direct recovery of Marshallian demand, although Hicksian demand, or the expenditure function, is needed for computing CV and EV. Hausman (1981) applies the duality theory and suggests a general method to recover the corresponding Hicksian demand from linear and log-linear Marshallian demand models. The method utilizes

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[2] In fact, P_H can be very large (approaching infinity) when the initial consumption is large. The discussion of the size of CV under the linear Marshallian demand model is discussed in the next section.

[3] Alternatively CV and EV can be derived by solving $U(P_H,I_0+CV)=U(P_0,I_0)$ and $U(P_0,I_0-EV)=U(P_M,I_0)$.

[4] Using the same notations, for the provision of a good, the initial utility level is $U_0=U(P_M, I_0)$ and the new utility level is $U_1=U(P_0, I_0)$. Keeping the welfare measures positive, the CV and EV can be defined as follows: CV = I_0 -

Shephard's lemma and the equality between Marshallian and Hicksian demands at the optimum, and integrates the optimal demand function to derive the utility and expenditure functions for welfare analysis of a given price change. In the case of a policy ban, the price change is endogenously determined and needs to be derived. In this note, four commonly used demand models are examined for welfare analysis of a policy ban. The four models are linear, semi-log, log-linear, and linear in income-price ratio.

$$X^M = \alpha + \beta P + \gamma I \tag{3}$$

$$X^M = e^{\alpha+\beta P+\gamma I} \tag{4}$$

$$X^M = e^{\alpha} P^{\beta} I^{\gamma} \tag{5}$$

$$X^M = \alpha + \delta(I/P) \tag{6}$$

The Greek letters α, β, γ, and δ are parameters. It is assumed that the law of demand holds and X is normal; i.e., $\beta<0$, $\gamma>0$, and $\delta>0$. Note that these models can be extended to include socioeconomic variables by replacing α with $Z\theta$, where Z is a row vector of individual socioeconomic characteristics and "one" for the intercept term, and θ is a column vector of parameters. Equation (6) is actually the demand model from the linear expenditure system that is commonly used in demand analysis. The procedure to derive CV and EV for a policy ban is outlined as follows.

(i) Following the Hausman approach, solve the differential equation $\frac{dI}{dP} = X^H = X^M = g(P,I)$ to derive the expenditure and indirect utility functions.

(ii) Apply Shephard's lemma to derive Hicksian demand function. (Alternatively, substitute the expenditure function into I in X^M to derive Hicksian demand function.)

(iii) Set X^H (X^M) to zero to derive the choke price P_H (P_M).

$I(P_0, U(P_M,I_0)) \equiv$ WTP and EV = $I(P_H, U(P_0,I_0)) - I_0 \equiv$ WTA.

(iv) Substitute P_H and P_M into the expression (1) and (2) to derive CV and EV.

The integrating constant in the general solution of the differential equation in step (i) is conveniently set to U_0, the initial level of the utility. Since welfare measures are invariant to monotonic transformation of the utility function, the integrating constant can be set to any cardinal utility index for the purpose of welfare analysis. Following the outlined procedure, the analytical results of the expenditure function, Hicksian demand, choke prices, initial and new levels of utility, CS, CV, and EV for each of the four demand models are summarized in Table 1.

In the cases of linear Marshallian demand and linear expenditure system, the choke prices P_M and P_H are finite and different; hence, CV is derived based on a price change different from CS. In these two cases, P_H is a function of the initial level of utility (U_0) and can be expressed as a function of the initial consumption level X_0, price P_0, and income I_0. In contrast, the choke prices under semi-log and log-linear Marshallian demands go to infinity; that is, the quantity demanded approaches zero as the price increases but it is never equal to zero. In these cases, the range of price changes associated with the elimination of the good is from P_0 to infinity--the same for both the Marshallian and Hicksian demands. Under all four demand models, the utility levels before and after the elimination of the good, CS, CV, and EV can all be expressed as functions of the initial quantity demanded, price, income, and parameters in the demand model.[5]

3. Closeness of Consumer Surplus to the Exact Welfare Measures

By examining the formulae, the CV and EV for the elimination of a good do not always

[5] Bockstael et al. (BHS, 1984) tabulate the CV and EV for a fixed price change (from P_0 to P_1) under three specifications of the Marshallian demand. In this paper, the formulae of CV and EV for the elimination of a good under the semi-log (log-lin) and log-linear demand cases are the same as setting X=0 and $P_1 \to \infty$ in the BHS report. The reason is that the choke prices for both Marshallian and Hicksian demands approach infinity in these two cases. All CS, CV, and EV are defined over the same range of changes in price.

exist. In the cases of linear and semi-log Marshallian demands, CV does not exist unless the initial quantity demanded (X_0) is less than $-\beta/\gamma$, while for EV to be defined in the log-linear case, the constant income elasticity of demand must be greater than $1-I_0/CS$. In contrast, the CS measures under all four specifications of the Marshallian demand are relatively easy to calculate so it is not surprising that CS is commonly used to approximate CV and EV.[6] However, the approximation might not be a good one. As pointed out in Bockstael and McConnell (1980), the Willig bounds degenerate when X=0 because the income elasticity tends to infinity at zero consumption. Further, the Willig bounds are derived under a fixed price change. In the case of eliminating or providing a good that involves different choke prices between X^M and X^H such as the case of linear Marshallian demand, the Willig method does not directly apply to bound CS because of the different ranges of integration for deriving CS and CV. The possible relationships between CS and the exact welfare measures are presented at the bottom of Table 1.

In the case of linear Marshallian demand, there is no analytical relationship between CS and the exact welfare measures except that EV≤CS≤CV. Under the condition that $X_0<-\beta/\gamma$, CS and EV are bounded above ($CS < -\frac{\beta}{2\gamma^2}$ and $EV < -\frac{\beta}{e\gamma^2}$, where e=2.71828). In contrast, CV is very large when the initial consumption is close to $-\beta/\gamma$; i.e., under the linear Marshallian demand model, the WTA for no consumption of the good can be very large.

For the semi-log case, there is an analytical relationship between CS and the exact welfare measures (Table 1). However, the closeness of CS to CV or EV depends on the value of γ and the size of CS. For example, if income elasticity is 1 (i.e., $\gamma I=1$), the Taylor series expansion of CV at $\frac{CS}{I}=0$ indicates that CS is close to CV when CS is small relative to income.

6 All discussion is under the general assumptions that the law of demand holds and X is normal.

In the log-linear case, as the constant income elasticity $\gamma \to 0$; i.e., no income effect, CS=CV=EV. Applying the mathematical fact that $\lim_{m\to\infty}\left(1+\frac{t}{m}\right)^m = e^t$, it can be shown that as $\gamma \to 1$,

$CV = I_0(e^{CS/I_0} - 1)$ and $EV = I_0(1 - e^{-CS/I_0})$. Similar to the case of semi-log, it can be shown by Taylor series expansion that CS is closer to CV or EV when CS-income ratio is small. Finally, the CV and EV from the demand function of the linear expenditure system are well defined but there is no clear analytical relationship between the exact welfare measures and CS.

CS is often used for measuring losses of eliminating a good. The closeness of CS to the exact welfare measures depends on the initial consumption level and parameters in the demand function. It is seen in this note that CS is not always a good welfare approximation and the exact welfare measures for a policy ban are readily derived for common Marshallian demand models. One caveat is that the exact welfare measures could be undefined under certain circumstances, which makes CS a convenient, yet possibly misleading, substitute. Empirically it is important to keep in mind the formulae of exact welfare measures in determining modeling strategies when welfare analysis is the ultimate goal.

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Table 1. Welfare Measures for the Elimination of a Good

Marshallian Demand (Initial $P=P_0$, $I=I_0$, $X=X_0$)	Linear $X^M = \alpha + \beta P + \gamma I$	Semi-Log $\ell n X^M = \alpha + \beta P + \gamma I$	Log-Linear $\ell n X^M = \alpha + \beta \ell n P + \gamma \ell n I$ ($\beta<-1$)	Linear Expenditure $X^M = \alpha + \delta(I/P)$ ($\alpha<0$)
Expenditure Function $I(P,U)$	$-\frac{1}{\gamma}\left(\alpha+\beta P+\frac{\beta}{\gamma}\right)+e^{\gamma P}U$	$-\frac{1}{\gamma}\ell n\left(-\gamma\left(\frac{e^{\alpha+\beta P}}{\beta}+U\right)\right)$	$\left(\frac{1-\gamma}{1+\beta}e^{\alpha}P^{1+\beta}+(1-\gamma)U\right)^{\frac{1}{1-\gamma}}$	$\frac{\alpha}{1-\delta}P+UP^{\delta}$
Hicksian Demand $\partial I(P,U)/\partial P$	$X^H=-\frac{\beta}{\gamma}+\gamma e^{\gamma P}U$	$-\frac{1}{\gamma}\frac{e^{\alpha+\beta P}}{\frac{1}{\beta}e^{\alpha+\beta P}+U}$	$e^{\alpha}P^{\beta}\left(\frac{1-\gamma}{1+\beta}e^{\alpha}P^{1+\beta}+(1-\gamma)U\right)^{\frac{\gamma}{1-\gamma}}$	$\frac{\alpha}{1-\delta}+\delta UP^{\delta-1}$
Choke Price for $X^M=0$ P_M	$-\frac{\alpha+\gamma I}{\beta}$	∞	∞	$-\frac{\delta I}{\alpha}$
Choke Price for $X^H(U_0)=0$ P_H	$\frac{1}{\gamma}\ell n\left(\frac{\beta}{\gamma^2 U_0}\right)=P_0-\frac{1}{\gamma}\ell n\left(1+\frac{\gamma}{\beta}X_0\right)^{*}$	∞	∞	$\left(-\frac{\alpha}{\delta(1-\delta)U_0}\right)^{\frac{-1}{1-\delta}}=\left(-\frac{\alpha}{\delta}\right)^{\frac{-1}{1-\delta}}P_0\left(\frac{I_0}{P_0}-X_0\right)^{\frac{1}{1-\delta}}$
Initial Level of Utility $U_0=U(P_0, I_0)$	$e^{-\gamma P_0}\left(\frac{X_0}{\gamma}+\frac{\beta}{\gamma^2}\right)$	$e^{-\gamma I_0}\left(-\frac{X_0}{\beta}-\frac{1}{\gamma}\right)$	$I_0^{1-\gamma}\left(-\frac{1}{1+\beta}\cdot\frac{P_0X_0}{I_0}+\frac{1}{1-\gamma}\right)$	$P_0^{-\delta}\left(I_0-\frac{\alpha}{1-\delta}P_0\right)=\frac{1}{1-\delta}P_0^{1-\delta}\left(\frac{I_0}{P_0}-X_0\right)$
New Level of Utility $U=U(P_M,I_0)$	$\frac{\beta}{\gamma^2}e^{\frac{\gamma}{\beta}(\alpha+\gamma I_0)}$	$-\frac{1}{\gamma}e^{-\gamma I_0}$	$\frac{1}{1-\gamma}I_0^{1-\gamma}$	$\frac{1}{1-\delta}\left(-\frac{\delta}{\alpha}\right)^{-\delta}I_0^{1-\delta}$
Consumer Surplus $CS=\int_{P_0}^{P_M}X^M(I_0)dp$	$-\frac{X_0^2}{2\beta}$	$-\frac{X_0}{\beta}$	$-\frac{P_0X_0}{1+\beta}$	$-P_0X_0+\delta I_0\ell n\left(-\frac{\delta}{\alpha}\frac{I_0}{P_0}\right)$
Compensating Variation (WTA) $CV=I(P_H,U(P_0,I_0))-I_0$	$-\frac{X_0}{\gamma}+\frac{\beta}{\gamma^2}\ell n\left(1+\frac{\gamma}{\beta}X_0\right)^{*}$	$-\frac{1}{\gamma}\ell n\left(1+\frac{\gamma}{\beta}X_0\right)^{*}$	$I_0\left(\left(1-\frac{1-\gamma}{1+\beta}\cdot\frac{P_0X_0}{I_0}\right)^{\frac{1}{1-\gamma}}-1\right)$	$\frac{1}{1-\delta}(-\alpha)^{\frac{-\delta}{1-\delta}}\left(\delta^{\frac{\delta}{1-\delta}}-\delta^{\frac{1}{1-\delta}}\right)p_0\left(\frac{I_0}{P_0}-X_0\right)^{\frac{1}{1-\delta}}-I_0$
Equivalent Variation (WTP) $EV=I_0-I(P_0,U(P_M,I_0))$	$\frac{X_0}{\gamma}+\frac{\beta}{\gamma^2}\left(1-e^{\frac{\gamma}{\beta}X_0}\right)$	$\frac{1}{\gamma}\ell n\left(1-\frac{\gamma}{\beta}X_0\right)$	$I_0\left(1-\left(1+\frac{1-\gamma}{1+\beta}\cdot\frac{P_0X_0}{I_0}\right)^{\frac{1}{1-\gamma}}\right)$	$\frac{1}{1-\delta}P_0\left(\frac{I_0}{P_0}-X_0\right)-\frac{(-\alpha)^{\delta}}{\delta^{\delta}(1-\delta)}I_0^{1-\delta}P_0^{\delta}$
Relationship between CS and CV	$CS<-\frac{\beta}{2\gamma^2}$, CV not bounded	$CV=-\frac{1}{\gamma}\ell n(1-\gamma\cdot CS)$	$CV=I_0\left(\left(1+\frac{CS/I_0}{1/(1-\gamma)}\right)^{\frac{1}{1-\gamma}}-1\right)$	--
Relationship between CS and EV	$CS<-\frac{\beta}{2\gamma^2}, EV<-\frac{\beta}{\gamma^2}e^{-1}$	$EV=\frac{1}{\gamma}\ell n(1+\gamma\cdot CS)$	$EV=I_0\left(1-\left(1-\frac{CS/I_0}{1/(1-\gamma)}\right)^{\frac{1}{1-\gamma}}\right)^{**}$	--

*The measure exists only if $X_0 < -\frac{\beta}{\gamma}$

**The measure is defined only if $\frac{1}{1-\gamma} > \frac{CS}{I_0}$ (i.e., $\gamma > 1 - \frac{I_0}{CS}$)

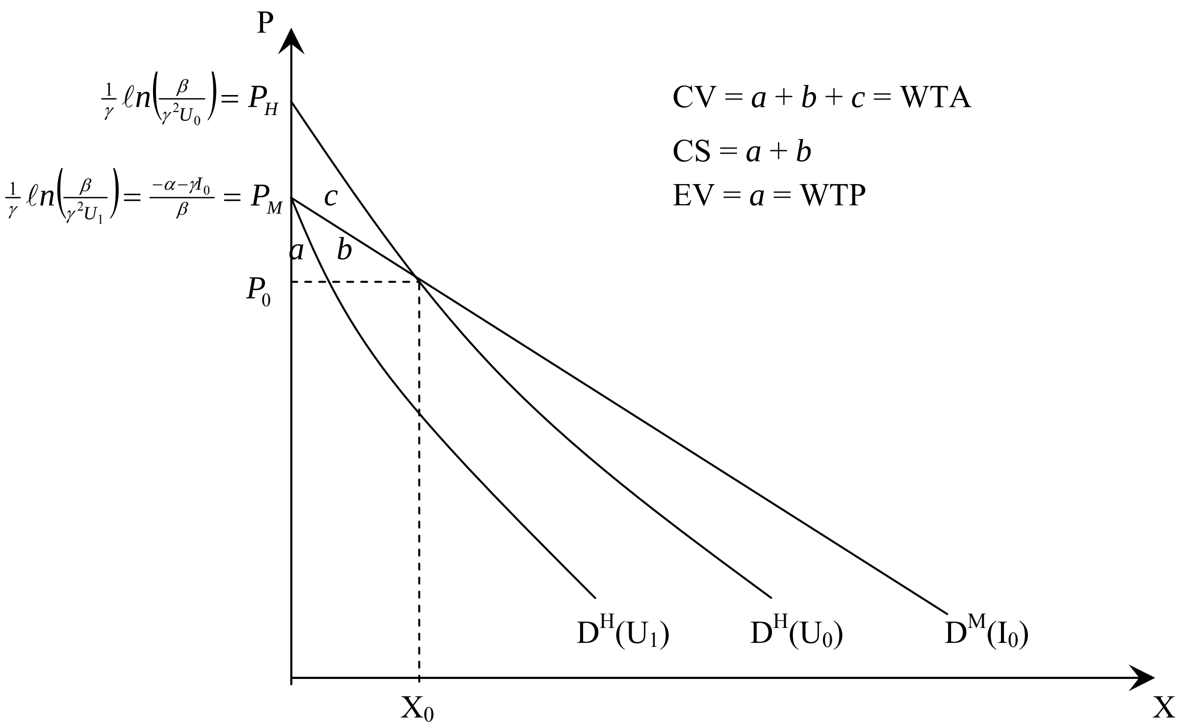

Figure 1. Linear Marshallian Demand and Corresponding Hicksian Demands ($X^M = \alpha + \beta P + \gamma I$)