Deriving Benefit Measures with Higher Precision: A Study of Economic Values of Air Quality

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Abstract

A calibration strategy using ridge regression to generate more precise estimates for a particular parameter in a regression model is proposed. Formulae to compute the proposed ridge estimates from standard OLS results are provided. Marginal effects of air pollution on property values for nineteen published studies are recomputed. Results show that ridge estimates are superior to the OLS estimates under the mean squared error criterion in all nineteen studies. The same strategy could be used to re-estimate key parameters of interest in other applications such as price elasticities for demand forecasts or the value of a statistical life from hedonic wage regressions.

Key words: Calibration of OLS Estimates, Economic Values of Air Quality, Hedonic Property Values, Adaptive Ridge Estimates

1. Introduction

One of the main concerns regarding the benefit measure of clean air derived from a hedonic housing model is that it varies widely. Smith and Huang (1995) review thirty-seven empirical hedonic property value studies and find that the benefit estimates range from zero to \$98.52 (in 82-84 dollars) for a one unit ($\mu g/m^3$) reduction in total suspended particulates (TSP). Some of the variation can be explained by the characteristics of different cities and the purposes of studies. However, the standard practice of including a large number of highly correlated regressors in the hedonic property value studies also reduces the estimation precision of the marginal benefit estimate of air quality. Multicollinearity is a common problem in the empirical hedonic property value studies, which does not result in biased estimation but the estimates can have large variances. It is also common in empirical economic analysis to focus on one or a few coefficient estimates. For example, the estimated price elasticities are used for demand forecasts, and the estimated wage premium of job risk from the hedonic wage regression can be used to estimate the value of a statistical life. From the policy standpoint, the precision of a particular coefficient estimate in these cases can be as important, if not more important, as the goodness of fit of the whole empirical model.

The purpose of this research is to explore alternative, more precise estimates of a particular coefficient *based on standard regression results*. In particular, the benefit estimates of air quality from hedonic property value regressions are studied. A simple calibration strategy to generate more precise estimates for a particular parameter in a regression model is proposed. Ridge regression, an alternative to OLS especially when multicollinearity is a concern, is employed as a calibration tool to derive more precise estimates.[1] Three estimators, the ordinary

[1] The use of ridge regression among economists has dwindled due to biasness of the associated estimators. When multicollinearity significantly influences the reliability of a point estimate, trading bias for smaller variances can be

least squares (OLS) estimator, the ridge estimator proposed by Huang (1999), and the traditional ridge estimator proposed by Hoerl, Kennard, and Baldwin (HKB, 1975) are compared. In addition to potential precision gains, an advantage of these ridge estimators is that they can be easily computed. This study provides simple formulae to calculate the two ridge estimates and their mean squared errors (MSEs) for a particular coefficient based on the standard OLS statistics. The new ridge estimates associated with the marginal effects of air pollution on property values are calculated based on results from nineteen published property value/air pollution studies and are compared with the original OLS estimates. It is found that the ridge estimators give more precise estimates than the OLS estimator in all nineteen studies.

2. Ridge Estimators

A multiple linear regression model has more than one explanatory variable. Suppose that the policy makers are particularly interested in the effect of one variable, X_1. Let Y be the dependent variable and X_2 be the set of (a-1) other independent variables (including the intercept column). Then, the linear model can be written as follows.

$$Y = X_1\beta_1 + X_2\beta_2 + \varepsilon \tag{1}$$

where ε is an nx1 vector of independent and identically distributed random errors that follow a joint normal distribution N(0, $\sigma^2 I$). I is the nxn identity matrix. Assume that $X_1'X_1$ and $X_2'X_2$ are non-singular. The OLS estimator of β_1 can be written as follows (Kmenta (1986), p. 398).

$$\hat{\beta}_1 = \left(X_1'M_2X_1\right)^{-1}\left(X_1'M_2Y\right) \tag{2}$$

where $M_2 = I - X_2\left(X_2'X_2\right)^{-1}X_2'$ and $X_1'M_2X_1 > 0$. Under the assumption that the model in

an effective way to improve estimation precision, as demonstrated in the case study of valuation of air quality below.

(1) is correctly specified, $\hat{\beta}_1$ is unbiased; thus its variance is also the MSE and is equal to $\sigma^2\left(X_1{}'M_2X_1\right)^{-1}$.

Hoerl (1962) suggests the following estimator: $\left(X'X+kI\right)^{-1}X'Y$, where k is a positive constant called the biasing parameter. The estimator is known as the ordinary ridge regression (ORR) estimator. The ORR estimator can be generalized by extending kI to some diagonal matrix with non-constant elements and the generalized ridge regression (GRR) estimator results (Hoerl and Kennard (1970)). In general, ridge estimators can significantly improve estimation precision but introduce bias. Precision-accuracy trade-off is the characteristic of ridge estimators.

In the case when the prime interest is a particular parameter in the model, say the coefficient of X_1, Huang (1999) proposes a ridge estimator that is derived from an MSE criterion: $\hat{\beta}_1^k=\left(X_1{}'M_2X_1+k\right)^{-1}\left(X_1{}'M_2Y\right)$. The bias and variance of $\hat{\beta}_1^k$ can be expressed generally as $-k\left(X_1{}'M_2X_1+k\right)^{-1}\beta_1$ and $\sigma^2\left(X_1{}'M_2X_1\right)\left(X_1{}'M_2X_1+k\right)^{-2}$, respectively. The corresponding MSE (= variance + bias^2) is $\left[\sigma^2\left(X_1{}'M_2X_1\right)+k^2\beta_1^2\right]\left(X_1{}'M_2X_1+k\right)^{-2}$. As shown in Huang (1999), the proposed ridge estimator $\hat{\beta}_1^k$ has a smaller MSE than the OLS estimator $\hat{\beta}_1$ for any positive k value if $\sigma^2-\beta_1^2\left(X_1{}'M_2X_1\right)$ is non-negative, or $\left(X_1{}'M_2X_1\right)\beta_1^2/\sigma^2\le 1$. The optimal choice of k can be determined by minimizing the MSE of $\hat{\beta}_1^k$ and it turns out to be $k^*=\sigma^2/\beta_1^2$.[2] Note that the criterion for selecting the optimal k value

[2]The detailed derivation of the optimal k^* is available upon request.

is based solely on minimizing the MSE of $\hat{\beta}_1$, which is different from the common MSE criterion in ridge regression that is based on minimizing MSE of estimates of *all* coefficients in the model. The optimal k^* does not depend on the parameters in the β_2 vector. Back substituting the optimal k into MSE, the expression of MSE at k^* is derived.

$$MSE(\hat{\beta}_1^*) = \frac{\sigma^2 X_1' M_2 X_1 + (k^*)^2 \beta_1^2}{\left(X_1' M_2 X_1 + k\right)^2} = \frac{\sigma^2}{X_1' M_2 X_1 + \frac{\sigma^2}{\beta_1^2}} \tag{3}$$

where $\hat{\beta}_1^*$ is the optimal ridge estimator of β_1 associated with k^*. Substituting $MSE\left(\hat{\beta}_1^*\right)$ for $MSE\left(\hat{\beta}_1^k\right)$ in the equation (2) in Huang (1999), it is clear that $\hat{\beta}_1^*$ is superior to $\hat{\beta}_1$ in terms of the MSE criterion as long as $\sigma^2 > 0$ and $\beta_1<0$.

Since the optimal k depends on unknown parameters, σ^2 and β_1, a natural solution is to substitute OLS estimates into k^*, denoted $\tilde{k}^* = \hat{\sigma}^2 / \hat{\beta}_1$. Let $\tilde{\beta}_1^*$ be the adaptive (or feasible) estimator of β_1 derived by substituting $\tilde{k}^*$ into $\hat{\beta}_1^k$ and it can be expressed as follows.[3]

$$\tilde{\beta}_1^* = \left(X_1' M_2 X_1 + \frac{\hat{\sigma}^2}{\hat{\beta}_1^2}\right)^{-1} X_1' M_2 Y \tag{4}$$

The adaptive ridge estimator $\tilde{\beta}_1^*$ is not a linear estimator. Its small and large sample properties are summarized in the Table 1 in Huang (1999).

One of the most applied adaptive ridge estimators is the one proposed by Hoerl, Kennard, and Baldwin (HKB, 1975). For comparison, this special case of GRR estimator is also examined. Hoerl, Kennard, and Baldwin applied the biasing parameter: $k_h = a\,\sigma^2 / \beta'\beta$ to deriving the ridge

[3]In this paper, the estimators are distinguished by different notations. The number of *'s indicates different ridge estimators. A ^ denotes a regular ridge estimator with non-stochastic k and a ~ implies a ridge estimator with a stochastic k (i.e., an adaptive ridge estimator).

estimator $\hat{\beta}_1^{**} = \left(X_1{}' M_2 X_1 + k_h \right)^{-1} X_1{}' M_2 Y$.[4] The mean square error of $\hat{\beta}_1^{**}$ can be shown as follows.

$$MSE(\hat{\beta}_1^{**}) = \frac{\sigma^2 X_1{}' M_2 X_1 + k_h^2 \beta_1^2}{\left(X_1{}' M_2 X_1 + k_h \right)^2} = \frac{\sigma^2 X_1{}' M_2 X_1 + \left(\frac{a\sigma^2}{\beta'\beta} \right)^2 \beta_1^2}{\left(X_1{}' M_2 X_1 + \frac{a\sigma^2}{\beta'\beta} \right)^2} \tag{5}$$

Substituting the estimator of the biasing parameter $\tilde{k}_h = a\,\hat{\sigma}^2 / \hat{\beta}'\hat{\beta}$ into $\hat{\beta}_1^{**}$, the HKB adaptive ridge estimator results.

$$\tilde{\beta}_1^{**} = \left(X_1{}' M_2 X_1 + \frac{a\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}} \right)^{-1} X_1{}' M_2 Y \tag{6}$$

The small and large sample properties of the HKB estimator can be found in Ullah et al. (1981) and Ullah et al. (1984). In contrast to the Huang estimator, $\tilde{\beta}_1^{*}$, which depends only on the OLS estimator of β_1, the HKB estimator is constructed based on the OLS estimators of *all* β's. Since both $\tilde{k}^{*}$ and $\tilde{k}_h$ are stochastic and depend on data, the adaptive estimators $\tilde{\beta}_1^{*}$ and $\tilde{\beta}_1^{**}$ do not guarantee a reduced MSE. Nonetheless, several Monte Carlo studies have shown reduction of MSE and enhanced stability by using adaptive ORR estimators especially when the problem of collinearity is severe (e.g., Hoerl, Kennard, and Baldwin (1975), Lin and Kmenta (1982), and Delaney and Chatterjee (1987)).

3. Recovering Ridge Regression Estimates from OLS Results

This section shows an important advantage of these ridge estimators--they can be easily

[4]For the selection of a data based *k* value from a more general family of adaptive ridge regression estimators, see Tracy and Srivastava (1992).

computed based on the statistics in the standard OLS output. Information commonly reported in empirical studies includes regression coefficient estimates, t values and/or standard errors of the point estimates, residual sum of squares, F value, and R^2. To recover $\tilde{\beta}_1^*$, $\tilde{\beta}_1^{**}$, and their corresponding MSEs from OLS results, two segments, $X_1' M_2 X_1$ and $X_1' M_2 Y$ must be expressed in terms of the OLS results. It turns out that $X_1' M_2 X_1$ and $X_1' M_2 Y$ can be expressed as follows.

$$\begin{aligned} X_1' M_2 X_1 &= \frac{\hat{\sigma}^2}{S_{\hat{\beta}_1}^2} \\ X_1' M_2 Y &= \hat{\beta}_1 \frac{\hat{\sigma}^2}{S_{\hat{\beta}_1}^2} \end{aligned} \tag{7}$$

where $\hat{\sigma}^2$ is the OLS estimate of σ^2 and $S_{\hat{\beta}_1}$ is the standard error of $\hat{\beta}_1$. Substituting (7) into the expressions of $\tilde{\beta}_1^*$ and $\tilde{\beta}_1^{**}$, the two ridge estimates can be easily calculated from the reported OLS results. The step-by-step derivations are given in Appendix A.

$$\tilde{\beta}_1^* = \frac{t^2}{1+t^2} \hat{\beta}_1 \tag{8}$$

$$\tilde{\beta}_1^{**} = \frac{\sum_{i=1}^{a} \hat{\beta}_i^2}{a S_{\hat{\beta}_1}^2 + \sum_{i=1}^{a} \hat{\beta}_i^2} \hat{\beta}_1 \tag{9}$$

where t is the t statistic for $\hat{\beta}_1$ and a is the number of parameters estimated in the original model. By examining the formulae, essentially the ridge estimators perform systematic adjustments to the OLS estimate to counteract the instability of the point estimate resulted from high correlation among regressors. The formula to compute the Huang estimator indicates that the less significant (i.e., smaller t statistic or larger standard error) the OLS estimate of β_1, the

more prominent is the adjustment (that the multiple in the formula is further away from 1). In contrast, the formula to compute the HKB estimator involves the sum of squares of all the OLS parameter estimates in the regression model. As a result, its adjustment is not as aggressive as the Huang estimator in addressing the imprecision in the estimation of β_1 because it takes into account all the parameter estimates to produce a more conservative weight that is closer to 1. The above formulae also make transparent that the ridge estimators always give more conservative measures than the OLS estimates.

The corresponding estimated MSEs are derived by substituting $\tilde{k}^*$, $\tilde{k}_h$, and $\hat{\beta}$ into the definition of $MSE\left(\hat{\beta}_1^*\right)$ and $MSE\left(\hat{\beta}_1^{**}\right)$ in (3) and (5). They can be written in terms of the OLS summary statistics as well.[5] (See Appendix A for the derivations.)

$$M\tilde{S}E(\tilde{\beta}_1^*) = \frac{\hat{\beta}_1^2}{S_{\hat{\beta}_1}^2 + \hat{\beta}_1^2} S_{\hat{\beta}_1}^2 = \frac{\hat{\beta}_1^2}{1+t^2} \tag{10}$$

$$M\tilde{S}E(\tilde{\beta}_1^{**}) = \frac{\left(\sum_{i=1}^{a} \hat{\beta}_i^2\right)^2 + a^2 S_{\hat{\beta}_1}^2 \hat{\beta}_1^2}{\left(aS_{\hat{\beta}_1}^2 + \sum_{i=1}^{a} \hat{\beta}_i^2\right)^2} S_{\hat{\beta}_1}^2 \tag{11}$$

The MSE formulae in (10) and (11) can also be separated into the bias and variance components, respectively.

$$Bi\tilde{a}s(\tilde{\beta}_1^*) = \frac{-S_{\hat{\beta}_1}^2}{S_{\hat{\beta}_1}^2 + \hat{\beta}_1^2} = \frac{-1}{1+t^2}\hat{\beta}_1 \qquad Va\tilde{r}iance(\tilde{\beta}_1^*) = \frac{\hat{\beta}_1^4}{\left(S_{\hat{\beta}_1}^2 + \hat{\beta}_1^2\right)^2} S_{\hat{\beta}_1}^2 = \frac{t^2\hat{\beta}_1^2}{(1+t^2)^2} \tag{12}$$

[5] The estimators of MSE presented in (10) and (11) are intuitive but they are not unbiased. The MSE of an estimator consists of variance and bias that its estimation requires the information of the first two moments of the estimator. As seen in Dwivedi et al. (1980) and Huang (1999), the exact first two moments of the HKB and Huang estimators are very complex, which makes it difficult to come up with an unbiased estimator for MSE. Srivastava and Giles (1991) derive the exact unbiased estimators for the first two moments of a special case of the GRR estimator to enable the unbiased estimation of MSE but the procedure requires numerical integration using original data and it cannot be computed based solely on the standard OLS statistics. The small sample properties of the MSE estimators in (10) and (11) are to be examined in future research.

$$\widetilde{Bias}(\widetilde{\beta}_1^{**}) = \frac{-aS_{\hat{\beta}_1}^2}{aS_{\hat{\beta}_1}^2 + \sum_{i=1}^{a}\hat{\beta}_i^2}\hat{\beta}_1 \qquad \widetilde{Variance}(\widetilde{\beta}_1^{**}) = \frac{\left(\sum_{i=1}^{a}\hat{\beta}_i^2\right)^2}{\left(aS_{\hat{\beta}_1}^2 + \sum_{i=1}^{a}\hat{\beta}_i^2\right)^2} S_{\hat{\beta}_1}^2 \tag{13}$$

When the classical assumptions hold, the OLS estimator of β_1 is unbiased (zero bias) with estimated variance equal to $S_{\hat{\beta}_1}^2$. As seen from the above formulae, the bias of a ridge estimator depends on the magnitude and significance (or variability) of the OLS point estimators, so does the potential reduction in variance. The overall gain of trading bias for smaller variance is indicated by the reduction in MSE

An alternative approach to derive ordinary ridge regression estimates is to run OLS on an augmented model (Vinod and Ullah (1981) which requires the original data. In addition, the covariance matrix of ridge estimates resulting from this procedure is incorrect and needs to be adjusted (Power and Bishop (1987)). The algebraic approach to calculating the new ridge estimates presented in this section utilizes existing OLS results and does not require regression reruns on original data.

4. Re-calculating Benefit Estimates of Air Quality from Published Hedonic Housing Studies

In this section, the ridge estimators are applied to re-examine the estimated impact of air pollution, specifically the amount of total suspended particulates (TSP), on property values in published studies in the past three and half decades. The formulae in (8) and (9) enable us to recalculate the coefficient estimates of the air quality variable. The nineteen property value studies selected for recalculation all report OLS results and include the total suspended particulates (TSP) as the measure of air pollution in the hedonic property value equation.[6] The

[6]There is a wide variety of hedonic property value/air pollution studies. In this paper, I focus the review on those including the most common air pollutant, namely the TSP. Within the published property value/TSP studies, some

complete citations of the nineteen published studies are given in Appendix B. The original, published OLS estimates of the TSP coefficient and the recomputed ridge estimates of the nineteen studies, and their corresponding MSEs are reported in Table 1.[7]

Comparing the HKB's ridge estimator $\tilde{\beta}_1^{**}$ and $M\tilde{S}E\left(\tilde{\beta}_1^{**}\right)$ to the OLS results. The improvement of $\tilde{\beta}_1^{**}$ over $\hat{\beta}_1$ appears to be small. Since $\tilde{\beta}_1^{**}$ depends on all coefficient estimates, it is affected by the measurement units of the explanatory variables and the model size. The relatively large estimates of coefficients dominate the value of $aS_{\hat{\beta}_1}^2$ in most studies and make the biasing parameter *k* close to zero. Hence, $\tilde{\beta}_1^{**}$ is fairly close to $\hat{\beta}_1$ with an average of two percent shrinkage in magnitude. Among the nineteen housing price-TSP studies, Giannias (1996) uses the unconventional inverse TSP as the explanatory variable in the price equation. The HKB estimate for the Giannias study shows a higher MSE than the OLS estimate. For the rest of the eighteen studies, the HKB estimates show a slight improvement in precision. The estimated average improvement of precision is four percent.

The Huang estimates show an average of twenty-one percent shrinkage on the original coefficient estimates that the Huang estimator gives rise to a more conservative measure of the impact of air pollution on property values. The estimated precision of the coefficient estimates is indicated by the corresponding MSEs. By examining the ratio $M\tilde{S}E\left(\tilde{\beta}_1^{*}\right)$ to $S_{\hat{\beta}_1}^2$, it is seen that

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of them do not report the coefficient estimates of explanatory variables other than it for the TSP (e.g., Zabel and Kiel, 2000; Chay and Greenstone, 2000) that the HKB estimates cannot be computed from the reported results. Some of them employ estimation methods other than the OLS such as the method of maximum likelihood with Box-Cox transformation (e.g., Bender et al. 1980; Sonstelie and Portney, 1980; Bender and Hwang, 1985; Stover and Leven, 1992) that the proposed algebraic approach for recovering ridge estimates does not apply and are excluded from the review.

[7]Note that the examined studies adopted various functional forms for the hedonic price equation. The coefficient of the air quality variable indicates the impact of air quality on property values. However, it is not necessary the implicit price of clean air.

the mean square errors of the ridge estimates are universally smaller than the variance of the original OLS estimates in all studies. Further, the smaller the *t* value, the more is the $\widetilde{\beta}_1^*$ shrunk toward zero and the larger is $M\widetilde{S}E\left(\widetilde{\beta}_1^*\right)$. That is, the precision of the estimated coefficient improves less when the original OLS estimate is less significant, which makes sense. In general, $\widetilde{\beta}_1^*$ gives more reliable estimates since the estimated mean square error is uniformly reduced. The MSE reduction varies across studies. On average, the MSE is estimated to reduce by twenty-one percent. Further, the estimated average reduction of variance of the Huang estimates is over thirty percent.[8]

Given that the focus is to recover the marginal effect of one particular regressor (in this case the air quality variable), the Huang estimator appears to outperform the HKB estimator and the OLS estimator in terms of the MSE criterion in all nineteen studies.

5. Concluding Remarks

Under ideal conditions when all classical assumptions hold and all explanatory variables contribute independently to the variation in the dependent variable, we prefer unbiased estimators. Note that the theoretical property of unbiasedness provides 'accuracy on average' but it does not guarantee 'accurate point estimate' from a random sample. When a policy decision relies heavily on a point estimate, and high multicollinearity is present to cause the point estimate of interest to vary significantly with model specification and samples, it is worthwhile exploring alternative estimators that are more efficient.

This study employs adaptive ridge estimators to derive a set of more precise estimates

[8] The estimated variance and bias of the ridge estimates for each study are computed based on the formulae in (12) and (13). Unlike variance, the relative size of the bias of the ridge estimate to the bias of the OLS estimate cannot be computed since the OLS estimator is assumed to be unbiased (zero bias). Nevertheless, to give a sense of the bias in the ridge estimates, the average ratio of the estimated bias of the Huang estimate to the original OLS estimate is 0.21.

associated with marginal air quality benefits. In addition to the potential gain in precision, the new estimates can be easily calculated from OLS results. In this application, ridge estimation can be viewed as a statistical method to systematically calibrate the OLS estimates to derive more reliable benefit measures. Two ridge estimators, proposed respectively by Huang (1999) and Hoerl, Kennard, and Baldwin (HKB, 1975), are applied to the first stage estimation of the hedonic property value models. It is found that the Huang estimator provides significant improvement in estimation precision. In contrast, the traditional HKB estimator presents small efficiency gain in this case. It is because the Huang estimator is derived to focus on estimating the particular regression parameter of interest, while the HKB estimator has more concern on the goodness of fit of the regression model. Given that the goal is to improve the estimation of one particular parameter in a regression model, the Huang estimator is shown to be superior to the HKB estimator in this case.

There are many situations that the strong policy interest lies on a particular regression coefficient such as estimating the price elasticity of demand, returns to scale in production, the wage differential due to job risk in a wage equation, and in this study the implicit price of air pollution. Using the proposed ridge estimators and the corresponding simple formulae to re-examine and re-estimate the parameter of interest with better precision from existing studies can provide additional information to policy makers in various fields without incurring much additional costs. They can certainly be computed along with the standard OLS estimates for comparison in new studies.

Note that the ridge estimates are more precise yet smaller than OLS estimates due to the shrinkage nature of ridge estimators. Carrying this result into the second stage estimation of the hedonic property value model, one may predict the willingness-to-pay for air quality to be

smaller. This implies that the benefit of improved air quality is measured more conservatively when applying ridge estimates and the choice of estimates can affect policy decisions. On the other hand, in situations when the OLS benefit estimates are perceived to be too high, ridge estimators may provide statistically sound calibration to produce alternative, more convincing estimates.

The biasing parameter, *k*, in the adaptive ridge estimators is a function of the sample data and thus stochastic. Simulation results (e.g., HKB, 1975) show that the HKB estimator has a probability greater than 0.5 of producing estimates with smaller MSE than least squares. The probability of dominance increases with the number of regressors, σ^2, and the severity of multicollinearity. In this study, it is seen that the corresponding MSE of the Huang estimates are smaller than the estimated variance of the OLS estimator in all nineteen published studies. Further investigation via simulation to examine the empirical dominance of the Huang estimator over the OLS estimator is warranted.

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TABLE 1. Comparison of OLS ($\hat{\beta}$), HKB ($\tilde{\beta}^{**}$), and Huang ($\tilde{\beta}^{*}$) Estimates

Studies (Chronologically ordered)	Location	OLS Results Coef	OLS Results Variance (MSE0)	HKB Estimates Coef	HKB Estimates MSE (MSE1)	$MSE\,1/MSE\,0$	Huang Estimates Coef	Huang Estimates MSE (MSE2)	$MSE\,2/MSE\,0$
Anderson & Crocker 71	Washington D.C.	-0.0610	1.01E-03	-0.0610	1.01E-03	0.9987	-0.0480	7.95E-04	0.7863
"	"	-0.1698	2.59E-03	-0.1686	2.55E-03	0.9862	-0.1558	2.38E-03	0.9176
"	"	0.0049	9.99E-04	0.0048	9.52E-04	0.9536	0.0001	2.34E-05	0.0235
"	"	-0.0302	7.56E-04	-0.0298	7.36E-04	0.9734	-0.0165	4.13E-04	0.5467
"	Kansas City	-0.0876	1.31E-03	-0.0875	1.31E-03	0.9984	-0.0748	1.12E-03	0.8541
"	"	-0.0641	8.41E-04	-0.0639	8.37E-04	0.9948	-0.0532	6.98E-04	0.8301
"	"	-0.1237	6.45E-04	-0.1222	6.32E-04	0.9797	-0.1187	6.19E-04	0.9595
"	"	-0.0972	1.20E-03	-0.0953	1.16E-03	0.9650	-0.0862	1.07E-03	0.8870
"	St. Louis	-0.1192	2.26E-03	-0.1190	2.25E-03	0.9973	-0.1029	1.95E-03	0.8630
"	"	-0.0627	1.32E-03	-0.0624	1.31E-03	0.9920	-0.0469	9.91E-04	0.7479
"	"	-0.1211	1.30E-03	-0.1195	1.26E-03	0.9754	-0.1113	1.19E-03	0.9188
"	"	-0.0827	1.62E-03	-0.0821	1.60E-03	0.9856	-0.0668	1.31E-03	0.8081
Deyak & Smith 74	Multiple SMSAs	-0.0870	1.96E-03	-0.0866	1.95E-03	0.9911	-0.0691	1.56E-03	0.7940
"	"	-0.0880	1.87E-03	-0.0875	1.85E-03	0.9883	-0.0709	1.51E-03	0.8050
"	"	-0.0870	1.84E-03	-0.0866	1.82E-03	0.9894	-0.0700	1.48E-03	0.8040
"	"	-0.0830	1.95E-03	-0.0827	1.94E-03	0.9930	-0.0647	1.52E-03	0.7790
"	"	-0.0850	1.80E-03	-0.0847	1.79E-03	0.9935	-0.0680	1.44E-03	0.8000
Smith & Deyak 75	Multiple Cities	-0.1150	4.22E-03	-0.1149	4.19E-03	0.9929	-0.0872	3.20E-03	0.7580
"	"	-0.0480	1.57E-03	-0.0480	1.57E-03	0.9990	-0.0285	9.35E-04	0.5940
Berry 76	Chicago	0.4150	5.89E-02	0.3362	4.46E-02	0.7580	0.3092	4.39E-02	0.7450
Polinsky & Rubinfeld 77	St. Louis	-0.132	1.94E-03	-0.1317	1.93E-03	0.9958	-0.1188	1.74E-03	0.9000
"	"	-0.137	1.16E-03	-0.1369	1.15E-03	0.9988	-0.1291	1.09E-03	0.9420
"	"	-0.14	2.03E-03	-0.1385	1.98E-03	0.9796	-0.1269	1.84E-03	0.9064
"	"	-0.149	1.09E-03	-0.1482	1.08E-03	0.9902	-0.1420	1.04E-03	0.9532
Harrison & Rubinfeld 78	Boston	-0.0510	4.10E-05	-0.0510	4.10E-05	0.9990	-0.0502	4.00E-05	0.9850
Smith 78	Chicago	-42.9100	8.55E+01	-42.6069	8.55E+01	0.9998	-41.0054	8.17E+01	0.9560
"	"	-51.0900	2.82E+02	-51.0768	2.82E+02	0.9995	-46.1015	2.55E+02	0.9020
Nelson 78	Washington D.C.	-0.048	8.41E-04	-0.0480	8.40E-04	0.9991	-0.0352	6.16E-04	0.7326
"	"	-0.078	6.76E-04	-0.0779	6.75E-04	0.9986	-0.0702	6.08E-04	0.9000
"	"	-0.116	7.29E-04	-0.1159	7.28E-04	0.9988	-0.1100	6.92E-04	0.9486
"	"	-0.093	7.29E-04	-0.0929	7.28E-04	0.9982	-0.0858	6.72E-04	0.9223
Brookshire, et al. 79	S. Ca. Air Basin	-316.890	1.30E+04	-316.890	1.30E+04	1.0000	-280.584	1.15E+04	0.8850
"	S. Ca. Air Basin	0.0000	0.00E+00	0.0000	0.00E+00	1.0000	0.0000	0.00E+00	0.9460
Jackson 79	Milwaukee	-1.4730	4.56E+00	-1.4730	4.56E+00	1.0000	-0.4751	1.47E+00	0.3230
McDonald 80	Chicago	0.0010	1.60E-05	0.0010	1.60E-05	0.9999	0.0001	1.00E-06	0.0590
"	"	0.0030	7.00E-05	0.0030	6.80E-06	0.0966	0.0017	4.00E-06	0.5690
"	"	-0.0020	9.00E-05	-0.0020	8.40E-06	0.0934	-0.0006	3.00E-06	0.3160
Brookshire, et al. 82	Los Angeles	-0.2218	3.36E-03	-0.2136	3.19E-03	0.9479	-0.2078	3.15E-03	0.9360
Palmquist 84	Atlanta	-45.4700	8.08E+01	-45.4690	8.07E+01	1.0000	-43.7608	7.77E+01	0.9620
"	Denver	-26.0400	1.31E+01	-26.0400	1.31E+01	1.0000	-25.5461	1.29E+01	0.9810
"	Houston	-11.8600	1.36E+01	-11.8600	1.36E+01	1.0000	-10.8168	1.24E+01	0.9120
"	Louisville	-1.6000	1.11E+01	-1.6000	1.11E+01	1.0000	-0.2996	2.08E+00	0.1870
"	Miami	-0.0030	0.00E+00	-0.0030	1.00E-07	0.9998	-0.0030	0.00E+00	0.9920

"	Oklahoma City	-0.2200	3.36E+00	-0.2200	3.36E+00	1.0000	-0.0031	4.77E-02	0.0140
"	Seattle	-8.5900	3.37E+01	-8.5900	3.37E+01	1.0000	-5.8975	2.31E+01	0.6870
Brookshire et al. 85	Los Angeles Co.	-0.0018	8.09E-08	-0.0018	8.00E-08	1.0000	-0.0018	7.89E-08	0.9757
"	Bay Area Co.	-0.004	9.21E-08	-0.0040	9.00E-08	1.0000	-0.0040	9.16E-08	0.9943
"	Los Angeles Co	-0.001	3.98E-08	-0.0010	4.00E-08	1.0000	-0.0010	3.83E-08	0.9617
"	Bay Area Co.	-0.004	1.68E-07	-0.0040	1.70E-07	1.0000	-0.0040	1.66E-07	0.9896
Atkinson & Crocker 87	Chicago	-97.0270	1.24E+02	-97.0267	1.24E+02	1.0000	-95.7684	1.22E+02	0.9870
Blomquist, et al. 88	Multiple Counties	-0.5344	3.37E-03	-0.5344	3.36E-03	0.9991	-0.5282	3.33E-03	0.9880
Graves, et al. 88	LA, Orange, Riverside, San Bernardino (CA)	-0.012	5.89E-07	-0.0120	5.90E-07	0.9997	-0.0120	5.87E-07	0.9959
"	"	-0.01	5.48E-07	-0.0100	5.50E-07	1.0000	-0.0099	5.45E-07	0.9946
Giannias 96	Houston	6701.21	1.13E+07	3337.12	1.41E+07	1.2473	5351.55	9.04E+06	0.7986
Chattopachyay 98	Chicago	-0.1529	2.10E-03	-0.1528	2.10E-03	0.9990	-0.1403	1.93E-03	0.9176
Average Improvement in Precision						0.9608			0.7935

Appendix A. Ridge Estimators Expressed in Terms of OLS Summary Statistics

The typical OLS summary statistics:

$$\begin{cases} \hat{\beta}_1 = \left(X_1{}' M_2 X_1 \right)^{-1} X_1{}' M_2 Y \quad \Rightarrow \quad X_1{}' M_2 Y = \hat{\beta}_1 \left(X_1{}' M_2 X_1 \right) = \hat{\beta}_1 \dfrac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}} \\ S^2_{\hat{\beta}_1} = \hat{\sigma}^2 \left(X_1{}' M_2 X_1 \right)^{-1} \quad \Rightarrow \quad X_1{}' M_2 X_1 = \dfrac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}} \\ t = \dfrac{\hat{\beta}_1}{S_{\hat{\beta}_1}} \end{cases}$$

We may express $\widetilde{\beta}_1^*$, $M\widetilde{S}E\left(\widetilde{\beta}_1^*\right)$, $\widetilde{\beta}_1^{**}$, and $M\widetilde{S}E\left(\widetilde{\beta}_1^{**}\right)$ in terms of $\hat{\beta}_1$, $S^2_{\hat{\beta}_1}$, and t.

$$\widetilde{\beta}_1^* = \left(X_1{}' M_2 X_1 + \frac{\hat{\sigma}^2}{\hat{\beta}_1^2} \right)^{-1} X_1{}' M_2 Y$$

$$= \left(\frac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}} + \frac{\hat{\sigma}^2}{\hat{\beta}_1^2} \right)^{-1} \hat{\beta}_1 \frac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}}$$

$$= \frac{\hat{\beta}_1^2}{S^2_{\hat{\beta}_1} + \hat{\beta}_1^2} \hat{\beta}_1 = \frac{\dfrac{\hat{\beta}_1^2}{S^2_{\hat{\beta}_1}}}{1 + \dfrac{\hat{\beta}_1^2}{S^2_{\hat{\beta}_1}}} \hat{\beta}_1 = \frac{t^2}{1+t^2} \hat{\beta}_1$$

$$M\widetilde{S}E(\widetilde{\beta}_1^*) = \frac{\hat{\sigma}^2 X_1{}' M_2 X_1 + \left(\dfrac{\hat{\sigma}^2}{\hat{\beta}_1^2} \right)^2 \hat{\beta}_1^2}{\left(X_1{}' M_2 X_1 + \dfrac{\hat{\sigma}^2}{\hat{\beta}_1^2} \right)^2} [\equiv \text{var}\,iance + bias^2]$$

$$= \frac{\hat{\sigma}^2 \dfrac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}} + \left(\dfrac{\hat{\sigma}^2}{\hat{\beta}_1^2} \right)^2 \hat{\beta}_1^2}{\left(\dfrac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}} + \dfrac{\hat{\sigma}^2}{\hat{\beta}_1^2} \right)^2} = \frac{\dfrac{1}{S^2_{\hat{\beta}_1}} + \dfrac{1}{\hat{\beta}_1^2}}{\left(\dfrac{1}{S^2_{\hat{\beta}_1}} + \dfrac{1}{\hat{\beta}_1^2} \right)^2} = \frac{\hat{\beta}_1^2}{S^2_{\hat{\beta}_1} + \hat{\beta}_1^2} S^2_{\hat{\beta}_1} = \frac{\hat{\beta}_1^2}{1+t^2}$$

$$\widetilde{\beta}_1^{**} = \left(X_1{}' M_2 X_1 + \frac{a\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}} \right)^{-1} X_1{}' M_2 Y$$

$$= \left(\frac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}} + \frac{a\hat{\sigma}^2}{\sum_{i=1}^{a} \hat{\beta}_i^2} \right)^{-1} \hat{\beta}_1 \ \frac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}}$$

$$= \frac{\sum_{i=1}^{a} \hat{\beta}_i^2}{aS^2_{\hat{\beta}_1} + \sum_{i=1}^{a} \hat{\beta}_i^2} \hat{\beta}_1$$

$$M\widetilde{S}E(\widetilde{\beta}_1^{**}) = \frac{\hat{\sigma}^2 X_1{}' M_2 X_1 + \left(\frac{a\hat{\sigma}^2}{\sum_{i=1}^{a} \hat{\beta}_i^2} \right)^2 \hat{\beta}_1^2}{\left(X_1{}' M_2 X_1 + \frac{a\hat{\sigma}^2}{\sum_{i=1}^{a} \hat{\beta}_i^2} \right)^2} [\equiv \text{var}\,iance + bias^2]$$

$$= \frac{\hat{\sigma}^2 \left(\frac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}} \right) + \left(\frac{a\hat{\sigma}^2}{\sum_{i=1}^{a} \hat{\beta}_i^2} \right)^2 \hat{\beta}_1^2}{\left(\frac{\hat{\sigma}^2}{S^2_{\hat{\beta}_1}} + \frac{a\hat{\sigma}^2}{\sum_{i=1}^{a} \hat{\beta}_i^2} \right)^2} = \frac{\frac{1}{S^2_{\hat{\beta}_1}} + \left(\frac{a}{\sum_{i=1}^{a} \hat{\beta}_i^2} \right)^2 \hat{\beta}_1^2}{\left(\frac{1}{S^2_{\hat{\beta}_1}} + \frac{a}{\sum_{i=1}^{a} \hat{\beta}_i^2} \right)^2}$$

$$= \frac{\dfrac{S^2_{\hat{\beta}_1} \left(\sum_{i=1}^{a} \hat{\beta}_i^2 \right)^2 + a^2 \left(S^2_{\hat{\beta}_1} \right)^2 \hat{\beta}_1^2}{\left(S^2_{\hat{\beta}_1} \right)^2 \left(\sum_{i=1}^{a} \hat{\beta}_i^2 \right)^2}}{\dfrac{\left(\sum_{i=1}^{a} \hat{\beta}_i^2 + aS^2_{\hat{\beta}_1} \right)^2}{\left(S^2_{\hat{\beta}_1} \right)^2 \left(\sum_{i=1}^{a} \hat{\beta}_i^2 \right)^2}} = \frac{\left(\sum_{i=1}^{a} \hat{\beta}_i^2 \right)^2 + a^2 S^2_{\hat{\beta}_1} \hat{\beta}_1^2}{\left(aS^2_{\hat{\beta}_1} + \sum_{i=1}^{a} \hat{\beta}_i^2 \right)^2} S^2_{\hat{\beta}_1}$$

Appendix B: Hedonic Property Value/Air Pollution (TSP) Studies

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