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FINAL REPORT

THE VALUATION OF THE LIFE SHORTENING ASPECTS OF RISK

Submitted to

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THE VALUATION OF THE LIFE SHORTENING ASPECTS OF RISK

ABSTRACT

This report reviews the theoretical models for valuing life and small changes in risk, appendix A. Particular emphasis is given to the life cycle models developed by Shepard and Zeckhauser (1984), Cropper (1982) and Arthur (1981). This methodology is expanded to include approaches for valuing one life table when compared to another, appendix B. Such changes would be one of the principle benefits of a health program that extended life expectancy. The use of life tables permit an examination of such issues as:

- the effects of age;
- the latency of the risk, and
- alternative causes of death (but only as it affects the life table).

This methodology also allows such related issues as the discount rate, wealth, and the degree of risk aversion to be examined. If specific assumptions are made regarding these variables, then as shown by Shepard and Zeckhauser, the value of life can be calculated explicitly, and as shown in appendix B, the consumer surplus associated with one life table when compared to another can also be determined. These results are demonstrated using three causes of death: cardiovascular disease, fatal cancer and motor vehicle fatalities. The life cycle model is a fairly new development and as indicated here, additional research is needed to verify its conceptual basis.

THE VALUATION OF THE LIFE SHORTENING ASPECTS OF RISK

Introduction

Recent economic research has made important contributions to the problem of valuing human life and the related problems of valuing small changes in the probability of death. Such techniques permit a better understanding of the factors that can alter the valuation of risk, and ultimately will lead to a more comprehensive and accurate assessment of those programs that, as one of their benefits, increase life expectancies.

The technical and ethical issues of valuing life are not, of course, without controversy. For some, even the concept of assigning a dollar value to life is offensive and any amount, regardless of size, cannot possibly measure the true worth of survival to an individual. In contrast, valuing a small change in the risk of death is often more acceptable. After all, people daily engage in activities that they know increase their risk, even if only a little. Example activities would be jay walking, speeding, failing to use a seat belt, swimming, taking risky jobs, etc. These choices have some value to the individual and often these values can be implicitly or explicitly estimated.

Strictly speaking, value of life is meaningful only in the context of a particular time or age interval. What is gained or lost is not so much a life, but years of life. A health program can merely postpone death and by doing so, extend a person's life expectancy. Technically, such health programs increase the survival probabilities that make up one's life table; and the value of improved survival probabilities would be a function of the magnitude of the change, when they occur (latency), as well as other variables such as a person's age, health status, and existing level of risk. The purpose of this paper is to review these technical issues and to develop procedures for valuing one life table when compared to another based on existing theoretical and empirical results.

This report is organized around two appendices entitled "A Review of Theoretical

Models of the Value of Human Life" and "The Calculation of Compensating and Equivalent Surplus for Two Life Tables, the Perfect Market Case." The results of these appendices are summarized in the text to follow. This report begins with some definitions and a simple economic model for valuing small changes in risk.

Willingness to Pay (WTP)

Initial attempts to establish a value of life assumed that such a value could be determined by calculating what society would lose should the person die. The most important economic consequence of death is usually the loss of future earnings, and so, value of life often has been calculated as the net present value of these earnings. Such techniques are referred to as the human capital approach and their most useful attribute is that they are conceptually simple and can be estimated easily. However, most economists now reject this approach, for a variety of reasons. For example, without some modification, the value of life for a retired person would be zero or possibly even negative. As Violette and Chestnut (1983) note:

> Although providing useful benchmarks, these (the human capital) approaches do not provide an estimate of the benefits to the individual of reducing or preventing health risks because they do not reflect the change in utility, or well-being, that would result from the change in risk of illness or death (emphasis added).

Human capital approaches measure a person's contribution to society. In contrast, many economists now believe that a better measure of the value of life can be estimated from how much an individual is willing to pay for improved life expectancy. Conceptually, value of life, as determined by willingness to pay (WTP), is calculated by first establishing a willingness to pay for a small reduction in risk, say one in a million during the next year. Now if this risk reduction is given to one million people, then, an average of one life would be saved during this period, Thus the value of this one life would be one million (the population size) times the average amount each person is willing to pay to obtain this small reduction in risk. For example, if each person were willing to pay two dollars for this reduction, then the value of life would be estimated as two million dollars. This approach establishes a statistical value of life that is based upon small changes in risk. It need not have any relationship with the amount a person might or even could pay to avoid his own certain and immediate death.

Because there is no market place where mortality risks are openly traded, bought or sold, estimates of what a person is willing to pay for a small reduction in risk will have large uncertainties. Typically such estimates are determined from questionnaires or from related markets, such as wages paid to people in risky jobs. Given the difficulties of the task, it is not surprising that empirically estimated values of life have varied considerably, and typically range from 0.4 to 7 million dollars (1982 dollars). See Violette and Chestnut (1983). Larger and smaller estimates are not unknown. Further, these estimates usually do not reflect the effects of a variety of variables that are known to alter a person's WTP.

Since willingness to pay is a subjective judgment, it is expected to and does vary considerably among individuals. Certainly a person's wealth affects willingness to pay. Other factors, such as age, health status, degree of risk aversion, latency of death and many more (see Violette and Chestnut (1983) for a more complete summary) will also affect an individual's willingness to pay. Thus, each individual will have a unique value of life -- although this value could vary depending upon the circumstances surrounding the death being discussed -- and over an entire population there will be a distribution of "values of life." Most benefit-cost studies assume a single value for everyone, rich and poor, young and old alike. The next section describes a simple model that is the foundation for much of the theoretical work done to date. See for example, Freeman (1979), Linnerooth (1979), Rosen (1981) and Violette and Chestnut (1983). Details of this model are presented in appendix A.

A Simple Model

One of the simplest models for estimating the value of life considers a single time period (the immediate future) where an individual has probability p of dying and (1-p) of living. If this person lives, he will enjoy the consumption of goods and services of the amount C. This consumption has a utility U(C) to the individual, and it is assumed C = 0 implies death and that U(0) = 0. Suppose the individual has an opportunity to "buy" a small reduction, δ, in the probability of death, so that it becomes $p-\delta$. The question is, what is the maximum amount, B_δ, this person would be willing to pay to reduce p to $p-\delta$. If he pays too much, then the increase in the likelihood of living will not be sufficient to compensate the individual for the reduction in consumption.

Economic theory says an individual would pay an amount B_δ if, after the transaction, the expected utility is at least as great or greater than before the transaction. That is, since

$$E(U) = (1-p)\,U(C) \tag{1}$$

is the expected utility before the transaction, and this expected utility is

$$E(U_\delta) = (1 - p + \delta)\;\; U(C - B_\delta)$$

after the transaction, B_δ is a worthwhile expenditure if

$$(1 - p)\,U(C) \leq (1 - p + \delta)\,U(C - B_\delta) \tag{2}$$

The <u>maximum</u> that a person would pay, B_{max}, solves equation (2) assuming equality, and the person's implicit statistical value of life is $V_l = B_{max}/\delta$, using the willingness to pay criterion discussed earlier. In the limit, as $\delta \to 0$, it can be shown that

$$V_l = \frac{dC}{dp}$$

$$= \frac{U(C)}{(1 - p)\, U'(C)} \qquad (3)$$

where U'(C) is the derivative of U with respect to C. For additional detail, see appendix A.

Even at this basic level, two important properties of the statistical value of life, V_l, can be established. First, V_l increases with p, the probability of death. Therefore, equation (3) would generally imply older people have a greater statistical value of life because of higher mortality rates, ceteris paribus. (But note that other things are not equal. In particular, an older person has a shorter expected life span.) If it is assumed that the utility function U(C) is a concave increasing function (i.e., risk averse) so that

$$U''(C) \leq 0 \leq U'(C), \quad C > 0,$$

then expression (3) also implies V_l increases with C. This implies a wealthy person will have a greater WTP than a poor person, as might be expected.

From a practical perspective, the expected utility given in equation (1) ignores several important factors that might be associated with valuing life. It does not allow for a bequest motive or the effects of insurance and annuities. Perhaps even more important, it only considers the problem of valuing a statistical death that is to occur in the immediate future. In actuality, of course, most health improvement programs will alter a person's survival probabilities over an entire life time, and so a multi-period approach is more relevant.

Appendix A addresses how this simple model can be generalized to include these and other factors. Of particular interest here are the multi-period models of Cropper

(1982), Shepard and Zeckhauser (1984) and Arthur (1981). These models use a life cycle approach to generalize the single-period model as summarized below.

Life Cycle Modeling

In order to model the effects of changes to a life table of survival probabilities over several years, it is necessary to broaden the notation. Thus let

- y_i be income in year i,
- C_i be consumption in year i,
- q_i^* be the survival probability through year i, and
- D be the subjective discount factor for time (i.e., D = 1/(1+d) where d is the discount rate)

where i = 1,...T, and T is the maximum number of years a person can live. As a generalization of the single period model, expected utility is given by the expression

$$E(U) = \sum_{i=1}^{T} D^{i-1} q_i^* U(C_i) \qquad (4)$$

where $U(C_i)$ is the utility of consuming C_i in year i. This formulation assumes that the expected utility is received at the beginning of the period. In some cases an ending period or middle of period assumption is more appropriate. This can be achieved by multiplying equation (4) (and all following equations) by D or $D^{1/2}$ as appropriate. The conclusions presented here also assume that the utility function in equation (4) has the form

$$U(C_i) = C_i^{\beta} , \qquad (5)$$

and that the individual has access to insurance and annuities (the perfect market case). Under these circumstances, Shepard and Zeckhauser (1984) show the present value of a statistical life in year t, conditional on being alive in year t, is given by the expression:

$$V_{tt} = \frac{\bar{C}(1-\beta)}{\beta} \sum_{i=t}^{T} q_i^* D^{i-t} + \sum_{i=t}^{T} q_i^* D^{i-t} y_i$$

where the amount consumed every year, C, is

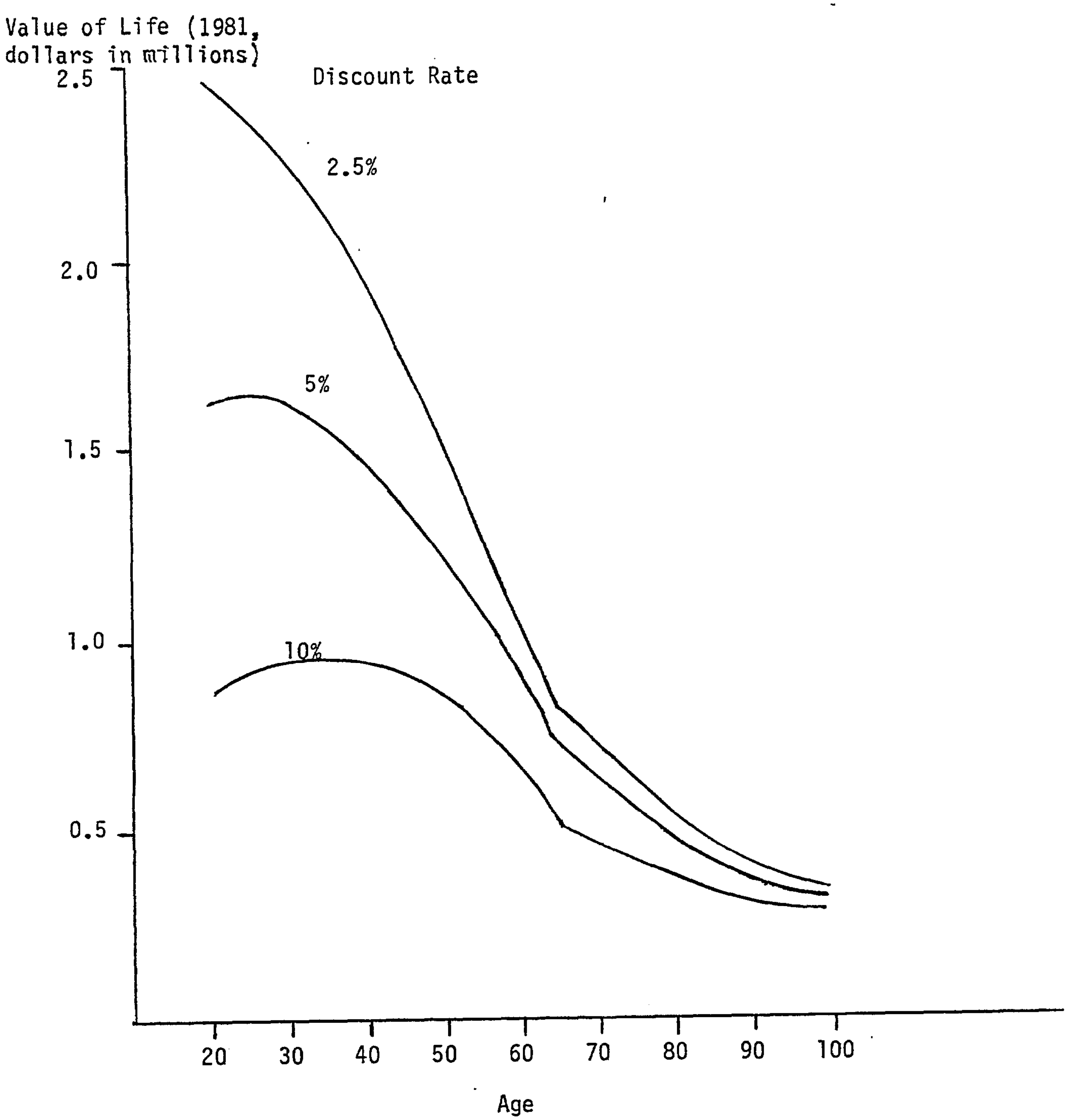
$$\bar{C} = \sum_{i=1}^{T} q_i^* D^{i-1} y_i \Bigg/ \sum_{i=1}^{T} q_i^* D^{i-1}, \qquad (7)$$

and is constant. (In this notation, V_{it} means the value of a statistical life conditional on being alive in year t discounted bock to year i.) When $t=1$, equation (6) simplifies to

$$V_{11} = \frac{\bar{C}}{\beta} \sum_{i=1}^{T} q_i^* D^{i-1}. \qquad (8)$$

Constant consumption is a consequence of the assumption that a person has access to annuities, has no bequest motive, and that the subjective discount factor for time, D, is equal to that for money. These issues are discussed in greater detail in appendix A. Figure 1 shows V_{tt} for males as calculated by equation (6) for three discount rates: 2.5%, 5% and 10%. (Other assumptions posited by Shepard and Zeckhauser are discussed later.) For the case that Shepard and Zeckhauser calculated where the discount rate is 5%, value of life is at a maximum at age 25 and has an estimated value of $1.64 million in 1981 dollars. (Unless specified, the remaining figures are in 1981 dollars.) This decreases to 0.31 million by age 100. The change in slope at age 65 is due to the Shepard and Zeckhauser assumption that the person has just retired, and income, y_i, is now zero

FIGURE 1. VALUE OF LIFE FOR MALES AS A FUNCTION OF THE DISCOUNT RATE



so that the second term in equation (6) drops out. When the discount rate is increased to 10% (not calculated by Shepard and Zeckhauser), the maximum value of life is reduced to 0.95 million dollars (1981) and it occurs at age 30. A discount rate of 2.5% (also not calculated by Shepard and Zeckhauser) implies a maximum value of life occurs at age 20 and is equal to 2.46 million dollars. Thus, the chosen discount rate can have a large effect on the estimated value of life, particularly in the early years. It also affects the age at which value of life is a maximum. In particular, it is shifted to younger ages as the discount rate decreases.

The values shown in figure 1 depend upon several assumptions that should be discussed a little more carefully. Shepard and Zeckhauser assume earnings followed the national pattern where, at age 20, the median income for males was 28% of the maximum etc., and the maximum occurred at age 50 and was equal to $24,000 (1981). It is easy to show that the value of life plotted in figure 1 scales directly with this maximum earning assumption. Thus a person earning twice as much at age 50 would have an estimated value of life twice as large. As Shepard and Zeckhauser observe:

> We assume that consumption in retirement is supplied only by savings accumulated during years of earnings. <u>Some factors are omitted in this assumption,</u> but may, as an approximation be treated as cancelling. For example, our earnings measure counts only money earnings, excluding employer-provided fringe benefits (insurance and pension contributions), transfer payments (Social Security benefits) and the value of home production; but work related expenses (commuting and meals away from home) and taxes (such as income and Social Security taxes) are also excluded. (emphasis added)

The parameter β in equation (5) calibrates the model for "risk aversion." A value of $\beta = 0.20$ was used for the example shown in figure 1. Since V_{11}, equation (8), scales directly with $1/\beta$, this parameter is clearly critical to the estimated value of life. For example, if β were in fact equal to 0.10 instead of 0.2, then V_{11} should be doubled. The effect of β on V_{tt}, $t > 1$, is more complicated, but V_{tt} should scale with $1/\beta$ at least approximately. In an attempt to assess the adequacy of their assumptions, Shepard and

Zeckhauser compared their results with the empirical findings of Fischer and Vaupel (1976). They concluded that at least for the subjects of that study, people were more risk averse than the parameter β = 0.20 implies (i.e., β is less than 0.2) and that these people also discounted future utility at a lower rate than 5%. Both of these conclusions, of course, would increase the estimated value of life as just discussed. Clearly more research is indicated.

Value of life, as estimated by equation (6) measures willingness to pay to avoid a <u>statistical deat</u>h that is to occur immediately. In actuality, most environmental programs affect survival probabilities over an individual's entire remaining lifetime, and a more useful measure, for benefit cost analysis, is one's willingness to pay for a new and better life table when compared to the current one. Appendix B develops the methodology to make this assessment, and the results are summarized below.

Compensating and Equivalent Surplus for Two Life Tables

Assume that an individual is endowed with a life table specified by survival probabilities $q_i^*(o)$, where o denotes "original." Under the assumption made earlier, the optimal consumption pattern is constant over years and is equal to C_o as calculated in equation (7). The expected utility, equation (4), therefore reduces to

$$E(U_o) = U(\bar{C}_o) \sum_{i=1}^{T} q_i^*(o) \; D^{i-1}$$

$$= U(\bar{C}_o) \; DLY(o) \; ,$$

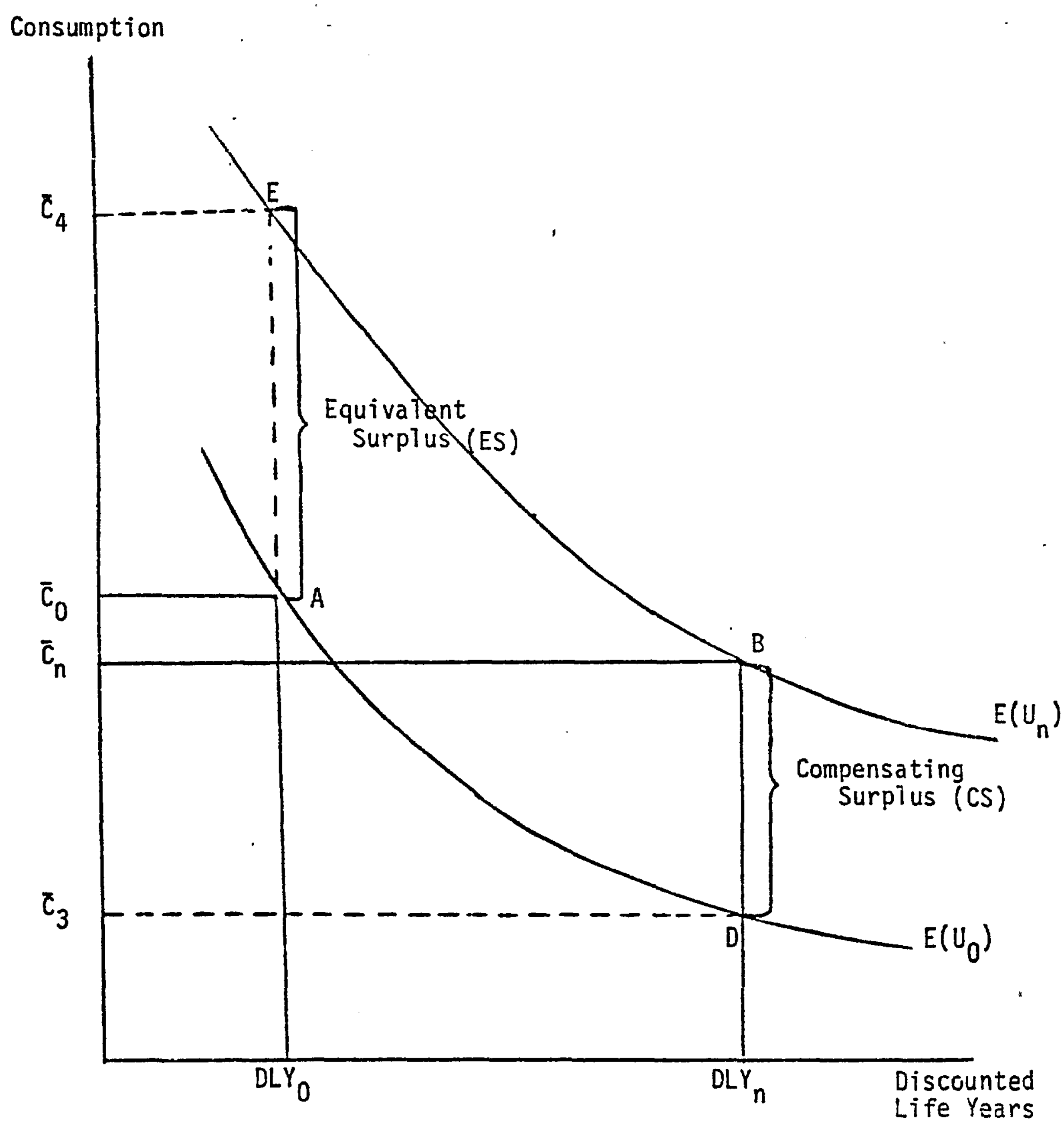
where DLY(o) is expected discounted life years, $\Sigma \, q_i^*(o)^{i-1}$, under the original life table. Now suppose a new life table becomes available, $q_i^*(n)$. This new table would change (presumably increase) the expected utility to

$$E(U_n) = U(C_n) \ DLY(n)$$

The basic question is what would a person pay to keep a new and improved life table when compared to the original table (sometimes called compensating surplus), or what amount of money would an individual demand to forego the new life table (equivalent surplus). Figure 2 illustrates these two calculations for an individual. The points A and B in figure 2 are determined by the two life tables $q_i^*(o)$ and $q_i^*(n)$. The distance B to D is the compensating surplus (CS) and represents the 'maximum amount of consumption a person with the new life table could pay annually and still be as well off (as measured by utility) as when he had the old life table. The distance A to E is the equivalent surplus (ES), and represents the amount of additional consumption a person with the old life table would need to be as well off as with the new life table at B.

Figure 3 shows three different survival probabilities (graphs of life tables) for males: an original endowed life table, survival probabilities that assume a cure for cancer has been found, and survival probabilities that assume all cardiovascular diseases have been cured. These last two tables were derived by S.H. Preston (1972) and (1976), and of course, represent only estimates of the effect should either of these two causes of death be eliminated. At age 20, the expected life span under the original table is 72.1 years. This increases to 82.4 years if cardiovascular diseases are cured. In contrast, a cure for cancer (neoplasms) would add 2.2 additional years to the expected life span giving a person age at 20 an expected life span of 74.3 years. Table 1 gives the incidence pattern for these causes of death. For example, of those that live to age 20, 59.3 percent will ultimately die of cardiovascular disease. From this group, 0.07 percent will die while age 20-24, etc. Table 1 shows that cardiovascular disease and cancer impact

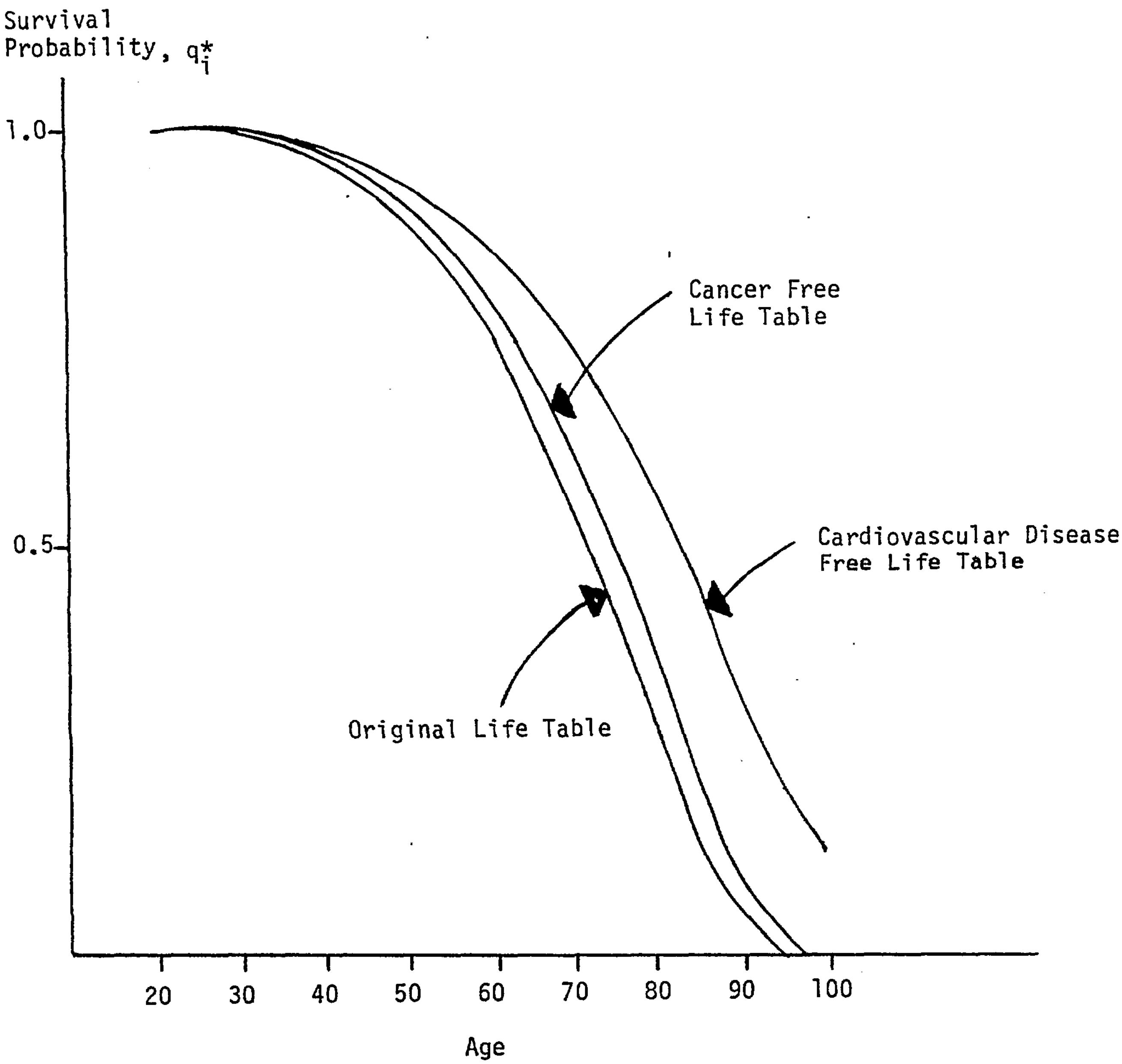
FIGURE 2. THE CALCULATION OF COMPENSATING AND EQUIVALENT SURPLUS



$$DLY = \sum_{t=1}^{T} D^{t-1} q_t^*$$

$$E(U) = U(\bar{C}) \cdot DLY$$

FIGURE 3. NOMINAL, CANCER FREE AND CARDIOVASCULAR DISEASE FREE LIFE TABLES FOR MALES



(1) Life tables as determined by S.H. Preston (1972) and (1976).

TABLE 1. AGE PATTERNS OF INCIDENCE FOR THREE CAUSES OF DEATH AMONG MALES (CONDITIONAL ON LIVING AT AGE 20)

Age	Cardiovascular Disease	Neoplasm	Motor Vehicles
20-24	0.07%	0.35%	17.10%
25-29	0.13%	0.49%	11.15%
30-34	0.29%	0.70%	8.73%
35-39	0.69%	1.11%	7.56%
40-44	1.48%	2.04%	6.98%
45-49	2.69%	3.57%	6.89%
50-54	4.51%	6.34%	6.94%
55-59	6.95%	9.73%	6.71%
60-64	9.55%	13.28%	6.22%
65-69	12.75%	16.16%	6.31%
70-74	14.50%	15.79%	5.64%
75-79	15.17%	13.57%	4.66%
80-84	14.34%	9.50%	3.36%
85+	16.87%	7.36%	1.75%
Fraction of all Deaths (age 20)	59.37%	16.22%	2.33%
Reference S.H. Preston (1972) and (1976).			

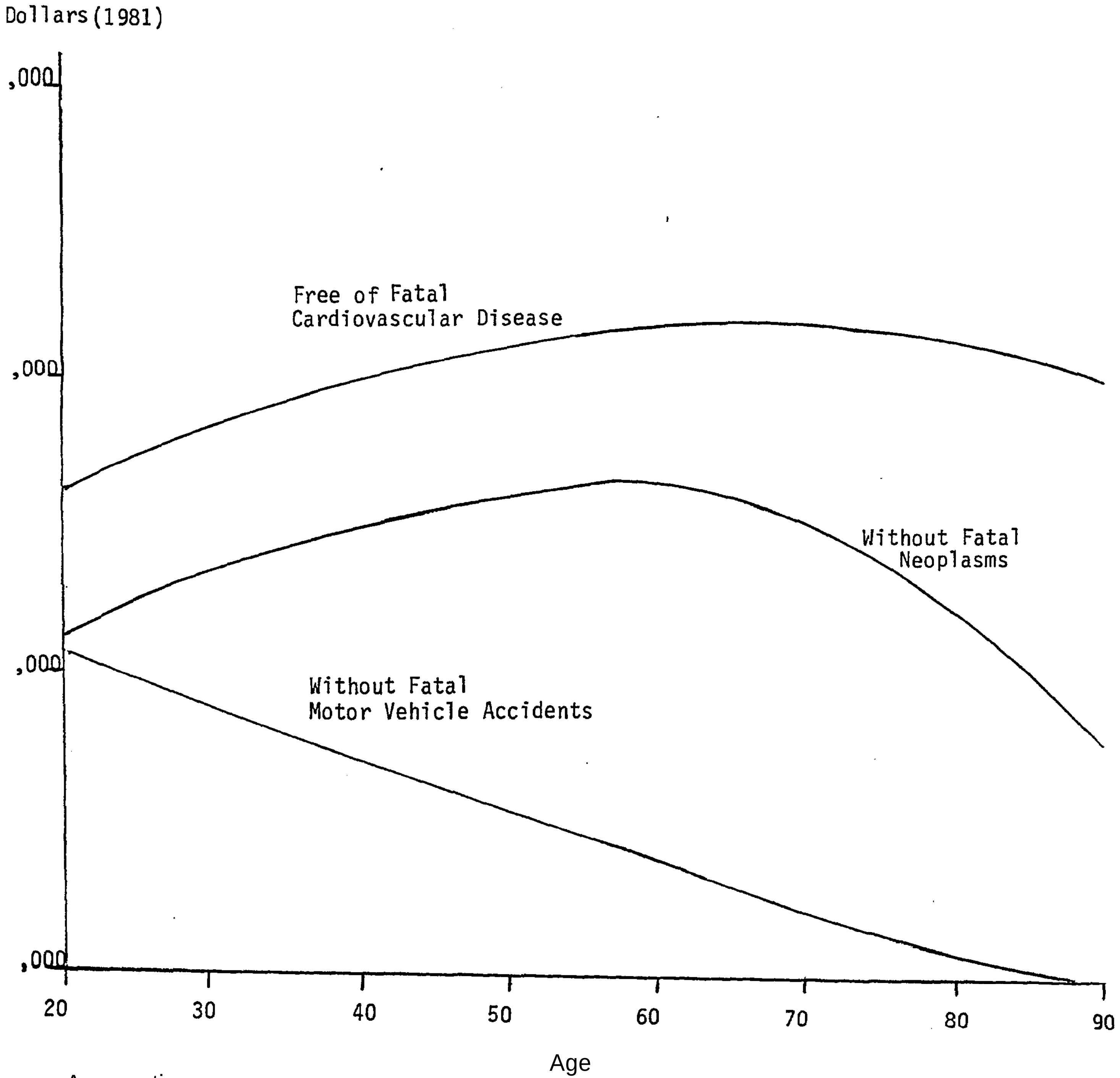
the elderly most. In contrast, risk of a motor vehicle fatality (also shown in table 1) is highest at age 20, and decreases slowly with age. Preston (1972) calculates that if motor vehicle fatalities were eliminated, a male life span would increase by only 0.6 years.

Appendix B develops the methodology for calculating compensating and equivalent surplus and illustrates these equations using the three new life tables just discussed. The results are plotted in figure 4 (log scale), using a discount rate of 5% and a risk aversion factor, β, of 0.20. For example, total compensating surplus (i.e., the sum of present and future payments discounted to the present) that a person would pay to keep a life table without cardiovascular disease is estimated to be nearly $45,000 at age 20. This increases to over $166,000 at age 65. These results show that such a life table would be highly valued, a result that is underscored by the fact that nearly 60% of all male deaths are caused by cardiovascular disease. As figure 4 shows, compensating surplus for a life table without fatal neoplasms or without motor vehicle fatalities is less, principally because the chance of dying from these causes is smaller. Also, in contrast to the other two life tables, the value of a life table without motor vehicle fatalities actually decreases with age. Equivalent surplus (not shown) is similar to compensating surplus except that equivalent surplus exceeds compensating surplus in all of these cases;[1] see appendix B for details.

Perhaps the most important property identified in this analysis is that the new life table will be most valued by that age group that is at the highest risk. While this has clear intuitive appeal, this property is not obvious from a simple examination of how "value of a statistical life" V_{tt} changes with age. In the examples used here, the value of statistical life was at a maximum at age 20 to 30, depending upon the discount rate, and for this reason one might assume these age groups would also have the highest willingness to pay for a new and better life table. This is not necessarily true. In fact, for

[1] This is due in part to the assumed utility function. Other utility functions could reverse this inequality.

FIGURE 4. COMPENSATING SURPLUS (EXPECTED NET PRESENT VALUE) FOR A LIFE TABLE FREE OF A PARTICULAR DISEASE/ACCIDENT TYPE CONDITIONAL ON BEING ALIVE AT AGE t: MALES



Assumptions:

Discount rate = 5%

β = 0.20

Maximum earnings = $24,000

cardiovascular diseases and neoplasms, willingness to pay for a new life table without these sources of death was a maximum in the age range of 55 to 85, when measured by either compensating or equivalent surplus.

The assumption that all neoplasms or cardiovascular diseases can be eliminated is unrealistic, particularly in the context of environmental programs that often improve survival rates only slightly (on the order of 1 in 100,000). Such a small change could increase an average life span only a few minutes or hours, and so willingness to pay would be correspondingly smaller. Table 2 presents willingness to pay estimates under the assumption that one person (not identified) in a hundred thousand receives a new life table (i.e., one person in a hundred thousand can be immunized against fatal cardiovascular disease, cancer or motor vehicle accidents). These values amount to only a few cents or at most a few dollars. However, if the environmental programs affected several millions of people, the overall value of the program would be substantial.

Since 59.37% of all male deaths are cardiovascular-related (at age 20), if

$$100{,}000/0.5937 = 168{,}435 \text{ twenty year old males}$$

received a life table that immunized one male in a hundred thousand, then the expected number of cardiovascular deaths would be reduced by one. Thus the value of a program that on average, prevented one cardiovascular death -- which is how many benefit cost studies pose the question -- would be valued as

$$168{,}435 \quad \text{x} \quad 0.54 \quad = \quad \$90{,}955. \tag{9}$$

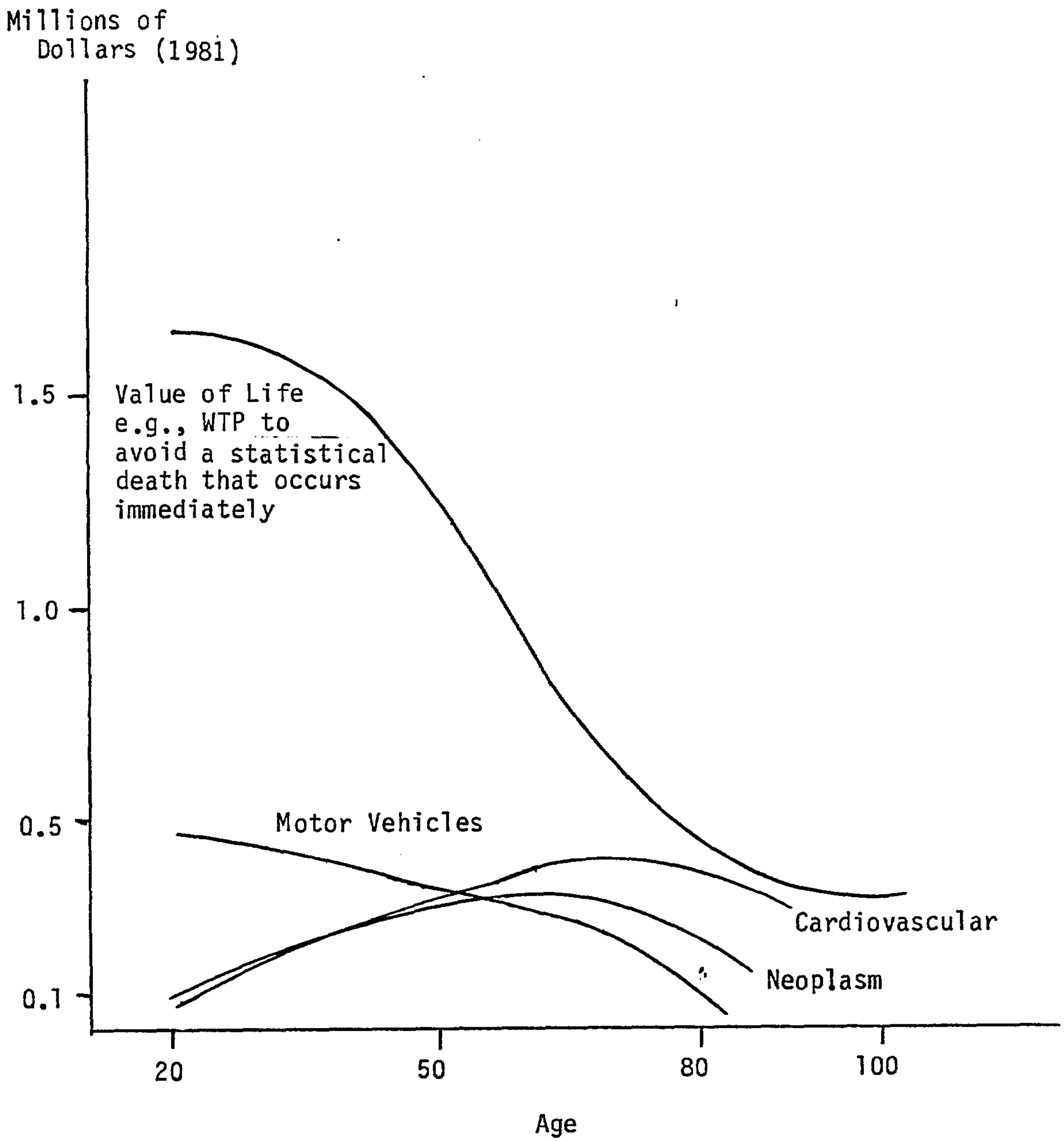
at age 20, using table 2. In contrast, under the same parameter assumptions, willingness to pay to avoid a statistical death (that occurs immediately) at age 20 is 1.62 million dollars. The difference is presumably due to the latency of the cardiovascular death. Figure 5 plots these and other values associated with a health program that could

TABLE 2. COMPENSATING SURPLUS (PER PERSON) ASSUMING ONE MALE IN A HUNDRED THOUSAND CAN BE IMMUNIZED AGAINST A SPECIFIC CAUSE OF DEATH

Age	Compensating Surplus (1981 dollars)* Cause of Death		
	Cardiovascular Disease	Cancer	Motor Vehicles
20	0.5463	0.1633	0.1149
25	0.7034	0.2106	0.0838
30	0.8895	0.2554	0.0760
35	1.1155	0.2987	0.0520
40	1.3800	0.3716	0.0512
45	1.6748	0.4408	0.0496
50	1.9495	0.4833	0.0355
55	2.2478	0.5357	0.0354
60	2.4950	0.5506	0.0350
65	2.6969	0.5022	0.0154
70	2.8457	0.4314	0.0169
75	2.8678	0.3247	0.0134
80	2.8036	0.2224	0.0083

*This is willingness to pay to immunize one male in a hundred thousand. It is not to be compared with willingness to pay to reduce the death rate by one in a hundred thousand, since immunization against a particular cause of death does not guarantee the person will not die of some other cause. See text. These calculations also assume a discount rate of 5% and a risk aversion factor, β_2 equal to 0.20.

FIGURE 5. WILLINGNESS TO PAY TO REDUCE THE NUMBER OF MALE DEATHS BY ONE, CONDITIONAL ON LIVING TO AGE t



Assumptions:

d Discount rate = 5%

β = 0.20

Maximum earnings = $24,000

eliminate exactly one cardiovascular, one cancer or one motor vehicle death. As would be expected, the value of avoiding an immediate death is always greater than the value of avoiding a (perhaps delayed) cardiovascular, cancer or motor vehicle death. In this regard, it is of interest to note that Litai (1980) using a very different methodology estimated that immediate risks require 30 times more compensation than delayed risks. The ratios shown in figure 5 typically vary from between 1 to 20, depending upon age and the source of the risk.

Discussion

As this report shows, the value of life or the value of one life table over another cannot be viewed as simply a single number. The usefulness of the economic analysis presented here is that it identifies how some of these factors might alter the estimated value of a statistical life and the value of one life table over another. Single-period models indicate that the value of a statistical life should:

- increase with wealth;
- increase with risk, and
- increase with risk aversion.

The multi-period models show that:

- value of a statistical life varies with age although the pattern varies with the discount rate and other assumptions,
- willingness to pay for a new life table is greatest for those age groups most at risk, and
- life saving programs that reduce latent risks have less value than programs that reduce immediate risks, _ceteris paribus._

If explicit assumptions concerning yearly income, y_i, consumption C_i, and the form of the utility function U are possible, then as the examples illustrate, the effects of these factors can be made fairly precise.

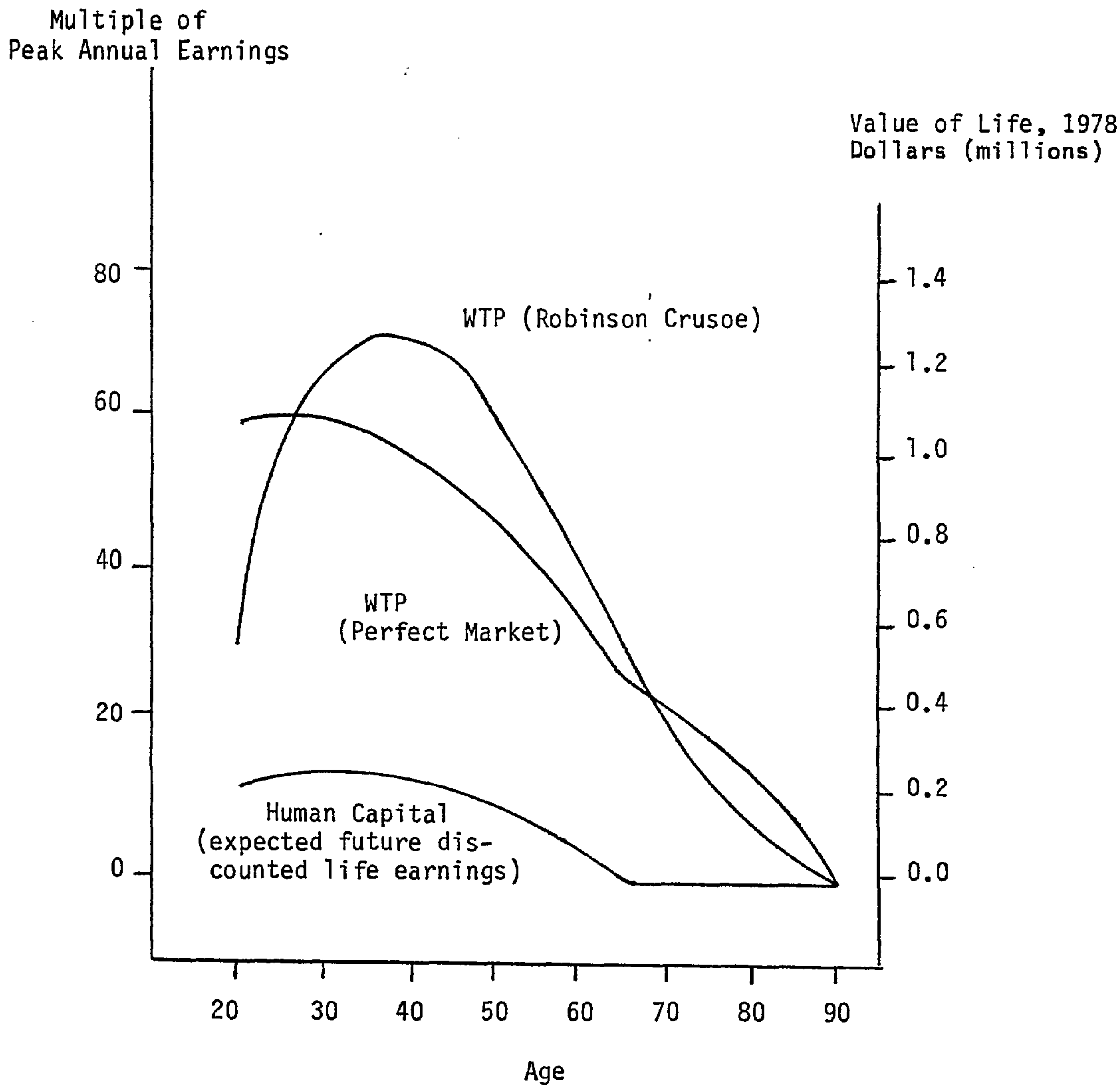
Shepard and Zeckhauser (1984) also consider a scenario they call the Robinson Crusoe case where it is assumed individuals do not have access to annuity markets and cannot borrow. Thus consumption is limited in the early years to the amount actually earned. Figure 6 compares these (in 1978 dollars) with those for the perfect market case. As might be expected, the value of a statistical life under the Robinson Crusoe case is less than that calculated under the perfect market case in the early years due to low income and the inability to borrow. Thus the effect of the perfect market assumption is to dampen the variability of the value of life over different age groups. Shepard and Zeckhauser believe that the Robinson Crusoe case and the perfect market case represent two poles, with actual willingness to pay for most people falling somewhere in between. Also shown in figure 6 is the expected future discounted life earnings that are normally associated with the human capital approach. Note that it is zero at age 65 and older due to the assumption that the individual has retired.

Future Research Efforts

In order for this methodology to be considered reliable enough to be used in EPA policy decisions several issues still need to be resolved. Certainly more work on the utility function is indicated. Shepard and Zeckhauser also express their interest in this subject.

> The objective of this analysis was to demonstrate the feasibility of a methodology and to indicated orders of magnitude, not to generate precise numbers for willingness to pay. If approach gains acceptance, substantial effort will have to be expended in estimating utility functions. Assessments of data about individual choices, as well as survey work, will be helpful in this task. Empirical research about utilities over lifetimes of different lengths would also be required to provide ultimate relevance. (emphasis added)

FIGURE 6. HUMAN CAPITAL AND WTP MEASURES OF VALUE OF LIFE AS FUNCTIONS OF AGE FOR AN AVERAGE MALE AS CALCULATED BY SHEPARD AND ZECKHAUSER



Assumptions: β = 0.2
Discount rate = 5%
Maximum earnings = $18,000

As figure 1 indicates, the chosen discount rate also has considerable leverage on the calculated value of life. In addition, all the examples presented here assumed the subjective discount rate for time equals the discount rate for money. If this is not true, then as Cropper shows, the optimal consumption pattern, C_i, will either increase or decrease with age under the perfect market scenario. No examples have been developed here that assume the subjective and monetary discount rates are different, however.

EPA uses a discount rate of 10% that presumably reflects the current interest rates for money. It is not clear if people would discount utility values at this same rate. Some researchers have, in fact, proposed models where utility is not discounted at all. Some insight into this problem could be acquired by an appropriately worded questionnaire and empirical research about lifetimes of different lengths. As indicated earlier, Shepard and Zeckhauser found some evidence that the subjective discount rate might be less than 5%, but additional work is necessary. In addition, the current life cycle models do not incorporate the effects of

- health status (quality of life, or QALYS),
- friends and family,
- nature of the risk (voluntary, the possibility of pain and suffering, etc.), and
- additional sources of income by age, such as fringe benefits, proprietor income, and transfer payments such as Social Security.

An important practical problem associated with the life cycle model is the requirement that the analyst be able to develop a life table for each proposed environmental change to compare with the reference table. For example, the cancer free life table developed by Preston would be inappropriate for cancers that mainly affected young people.

Implications for Benefit/Cost Studies

It is not uncommon to see a risk assessment conclude that if certain specified steps are taken (action levels, clean-ups, etc.) then x number of cancers, or other causes of death, can be prevented. Such summary statements are useful because (although the very real uncertainties of such assessments are often ignored) they

- provide an assessment of the effects of a particular action that is easy to interpret and
- provided there is an acceptable WTP estimate of value of life, such assessments can often be monetized for purposes of benefit cost analysis.

However, as this report has demonstrated, people should value life differently depending upon whether it is immediate or not. Further, the age distribution of those affected by the program needs to be factored in. The correct valuation of a program that a reduced number of cancer deaths has as one of its benefits should reflect this variation.

To illustrate, suppose an environmental hazard has been identified that jeopardizes 15 million males. In particular, this hazard has increased the risk of cancer by 1 in 100,000 over a lifetime of 72 years. Assuming the cancer is always fatal, a program that removes this hazard will cause the number of male cancer deaths to decrease by approximately

$$\frac{15,000,000}{72 \times 100,000} = 2.08$$

per year. Since these 2.08 deaths are reasonably immediate, (i.e., they occur within a year's time) their value could be estimated using value of life as calculated by equation (6), V_{tt} (also given in table 1 of appendix B) and graphed in figure 1. To account for age, the distribution of cancer deaths by age groups, table 1, can be used to calculate a weighted average. That is, if f_t is the fraction of all cancer deaths that occur in a particular age bracket in table 1, the average value of life of those who have died would be

$$V = \Sigma f_i V_{ii}$$
$$= \$793{,}900 \quad .$$

when V_{ii} is calculated assuming a discount rate of 5%. Thus 2.08 deaths would be worth 2.08 x 793,900 = 1.65 million dollars. Adding together the value of these deaths, for all future years and discounting to the present at 5% (this approach is essentially due to Arthur (1981)), this program would be valued at

$$1.65 \sum_{i=1}^{\infty} D^{i-1} = 33.0 \text{ million dollars.}$$

Note a similar calculation should be made for females if appropriate. Although there are some differences, as demonstrated in appendix B, 15 million females would value such a program approximately the same, *ceteris paribus*.

While the procedure just developed seems reasonable, an alternate, and possibly a more dependable approach for measuring the worth of this program would be to calculate the compensating surplus associated with the new life table for each of the 15 million people affected. Assuming a life table that prevents one cancer in a hundred thousand can be approximated by assuming 6.2 males[2] get a neoplasm free life table and 100,000 - 6.2 = 99,993.8 males get the old life table, then the compensating surplus for a 20 year old would be \$1.01 (6.2 x 0.1633, table 2). Assuming the males affected were typical of the general population, 11.32% of the 15,000,000 people should fall into the 20 to 24 age bracket (table 18 appendix B), the consumer surplus for these 1.70 million people would be calculated as 1.70 x 1.01 = 1.72 million dollars. Consumer surplus for other age groups

[2]Since a 20-24 year old has a 16.22% chance of dying from cancer, table B18, 1/.1622 = 6.2 males need to be immunized, on average, to avoid one cancer death.

are estimated in a similar fashion. In total, the compensating surplus for all 15 million sums to 33.8 million dollars, nearly the value calculated using the first technique. Table 3 compares these two methods for all three sources of death. These examples indicate that the two methods will produce comparable results, although the compensating surplus approach appears to give slightly higher values. It is interesting to note that a reduction in the number of motor vehicle fatalities is assessed as having a higher value than reductions for either cardiovascular or fatal neoplasms, presumably because motor vehicle fatalities affect young people most and young people have the greatest value of life. It is also worthwhile pointing out that the first method considered a time span from the present to the indefinite future, (albeit discounted at 5%), while the compensating surplus approach valued the new life table for only those presently living. This second approach therefore assumes the new life table will become the endowed life table for future generations and therefore has no associated compensating surplus.

Summary

This report has reviewed relevant economic models for valuing life (also see Appendix A) and discussed in more detail the life-cycle model of Arthur (1981) and Cropper (1982) and Shepard and Zeckhauser (1984). This model can be used to develop estimates of the value of life as a function of age and the value of a new life table over the original. Several examples have been used to illustrate the methodology. Considerable additional work is required to validate the approach and some research topics have been suggested.

TABLE 3. ALTERNATIVE CALCULATIONS OF THE BENEFITS
ASSOCIATED WITH A PROGRAM THAT REDUCES RISK
OF DEATH FROM A PARTICULAR CAUSE
BY 1 IN 100,000 AMONG 15 MILLION MALES, 1981 DOLLARS

Method	Cardiovascular Disease	Fatal Neoplasms	Motor Vehicle Fatalities
Weighted Average of Value of Life, Males	29.0 million	33.0 million	52.6 million
Total Compen-sating Surplus for 15 million Males	37.9 million	33.8 million	54.8 million

Assumptions: $\beta = 0.20$
Discount rate = 5%
Maximum earnings = $24,000

REFERENCES

1. Arthur, W.B. (1981). "The Economics of Risk to Life," American Economic Review, pp. 54-64.

2. Cropper, M.L. (1982). "Valuing Risks to Life"' preliminary report.

3. Fischer, G.W. and J.W. Vaupel (1976). "A Life Span Utility Model: Assessing Preference for Consumption and Longevity." Working Paper, Durham, NC: Center for Policy Analysis, Institute of Policy Sciences and Public Affairs, Duke University.

4. Freeman, M. (1979). The Benefits of Environmental Improvement. Resources for the Future, Inc. Johns Hopkins University Press.

5. Linnerooth, J. (1979). "The Value of Human Life: A Review of the Models," Economic Inquiry Vol. XVII, pp. 52-74.

6. Litai, D. (1980). "A Risk Comparison Methodology for Assessment of Acceptable Risk." Ph.D. Dissertation, MIT.

7. Pliskin, J.S., D.S. Shepard and M.C. Weinstein (1980). "Utility Functions for Life Years and Health Status," Operations Research 28, pp. 206-224.

8. Preston, S.H. (1976). Mortality Patterns in National Populations. Academic Press,

9. Preston, S.H., N. Keyfitz and R. Schoen (1972). Causes of Death: Life Tables for National Populations. Seminar Press, New York.

10. Rosen, S. (1981). "Valuing Health Risk," American Economic Review 71, pp. 241 - 245.

11. Shepard, D.S. and R.J. Zeckhauser (1984). "Survival Versus Consumption," Management Science Vol. 30, pp. 423-439.

12. Shepard, D.S. and R.J. Zeckhauser (1982). "Life Cycle Consumption and Willingness to Pay for Increased Survival." The Value of life and Safety, M.W. Jones-Lee ed. North-Holland Publishing Company.

13. Violette, D.M. and L.G. Chestnut (1983). Valuing Reductions in Risk: A Review of the Empirical Estimate.

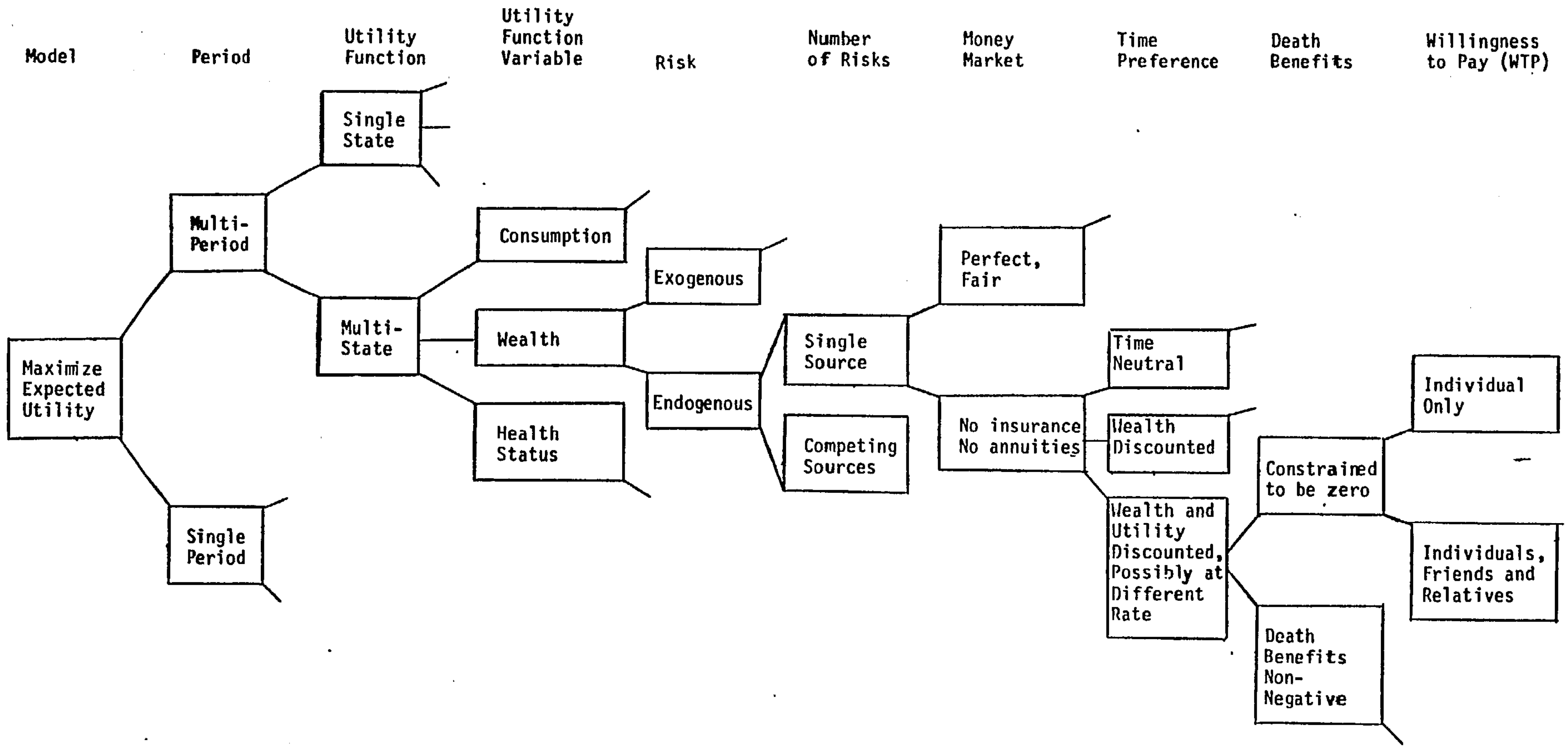
APPENDIX A: A REVIEW OF THEORETICAL MODELS OF THE VALUE OF HUMAN LIFE

The purpose of this appendix is to summarize current models for valuing human life and to discuss their properties with regard to such factors as age, degree of risk, latency of risk and life years. While the general principles of how to value life are now fairly well established--using willingness to pay (WTP)--there is still no consensus about details. Thus models in the literature will vary with regard to single vs multi-period analysis, the use of insurance and annuities, time preference assumptions, bequest motives, and other factors that might influence an individual's WTP. Figure 1 summarizes these choices and gives a taxonomy of various factors that have been examined in the value of life literature. Many of these factors will be discussed in later sections.

Theoretical models that calculate a value of life are important for several reasons. A theoretical approach imposes a logical rigor that helps discipline and define key concepts. Further, the need to make explicit assumptions concerning the behavioral characteristics of humans identifies important topics for empirical research. Finally, even after all the caveats and limitations have been acknowledged, some results may be robust to assumptions, and therefore be indicative of a more fundamental truth. All these attributes can contribute to EPA's research program for improving estimates of the value of life.

The next section describes a simple value of life model that ignores most of the factors given in figure 1. This model will serve as a control as other models of increasing complexity are introduced and discussed. It will be assumed here that the reader is already familiar with willingness to pay concepts and other relevant economic theory; and so this presentation will focus on the key properties of the model. Later sections

FIGURE I. A TAXONOMY FOR VALUE OF LIFE CALCULATIONS*



*not all combinations are possible.

will discuss more realistic, and therefore more complicated, models and address some of the questions and problems posed by EPA with regard to this work effort.

A Simple Model for Valuing Life

A simple model for estimating value of human life considers a single time period where an individual has a probability p of dying and (1-p) of living. If this person lives, he will enjoy the consumption of goods and services of the amount C. This consumption will have a utility U(C) and it is assumed C = 0 implies death and U(0) = 0. A person's expected utility is therefore given by

$$E(U) = (1-p)\ U(C).$$

The value of life V, as measured by willingness to pay, can be shown to be equal to dC/dp. This can be calculated by taking the total differential of E(U(C)), setting it equal to zero, and solving for dC/dp. Thus,

$$\begin{aligned} dE(U(C)) &= \frac{\partial\ E(U(C))}{\partial p}\ dp + \frac{\partial\ E(U(C))}{\partial C} \\ &= -\ U(C)dp + (1-p)\ U'(C)dC. \end{aligned}$$

So

$$\begin{aligned} V_l &= \frac{dC}{dp} \\ &= \frac{U(C)}{(1-p)\ U'(C)} \end{aligned} \qquad \text{(A1)}$$

This expression has been obtained by a number of people: Freeman (1979), Linnerooth (1980), Thaler & Rosen (1975) and others. Even at this basic level, several important properties of V_l can be established. For example, V_l increases with p, the probability of death, and therefore would generally assign higher values of life to older people because they have higher mortality rates, <u>ceteris paribus.</u> By assuming U(C) is a concave increasing function (i.e., risk averse) so that

$$U''(C) < 0 < U'(C), \tag{A2}$$

then expression (A1) above also will increase with the level of consumption, C. This implies that wealthy people will have a greater willingness to pay than poor people. In fact, if it were assumed that U(C) had the functional form U(C) = C^{β}, where $0 < \beta \leq 1$, then

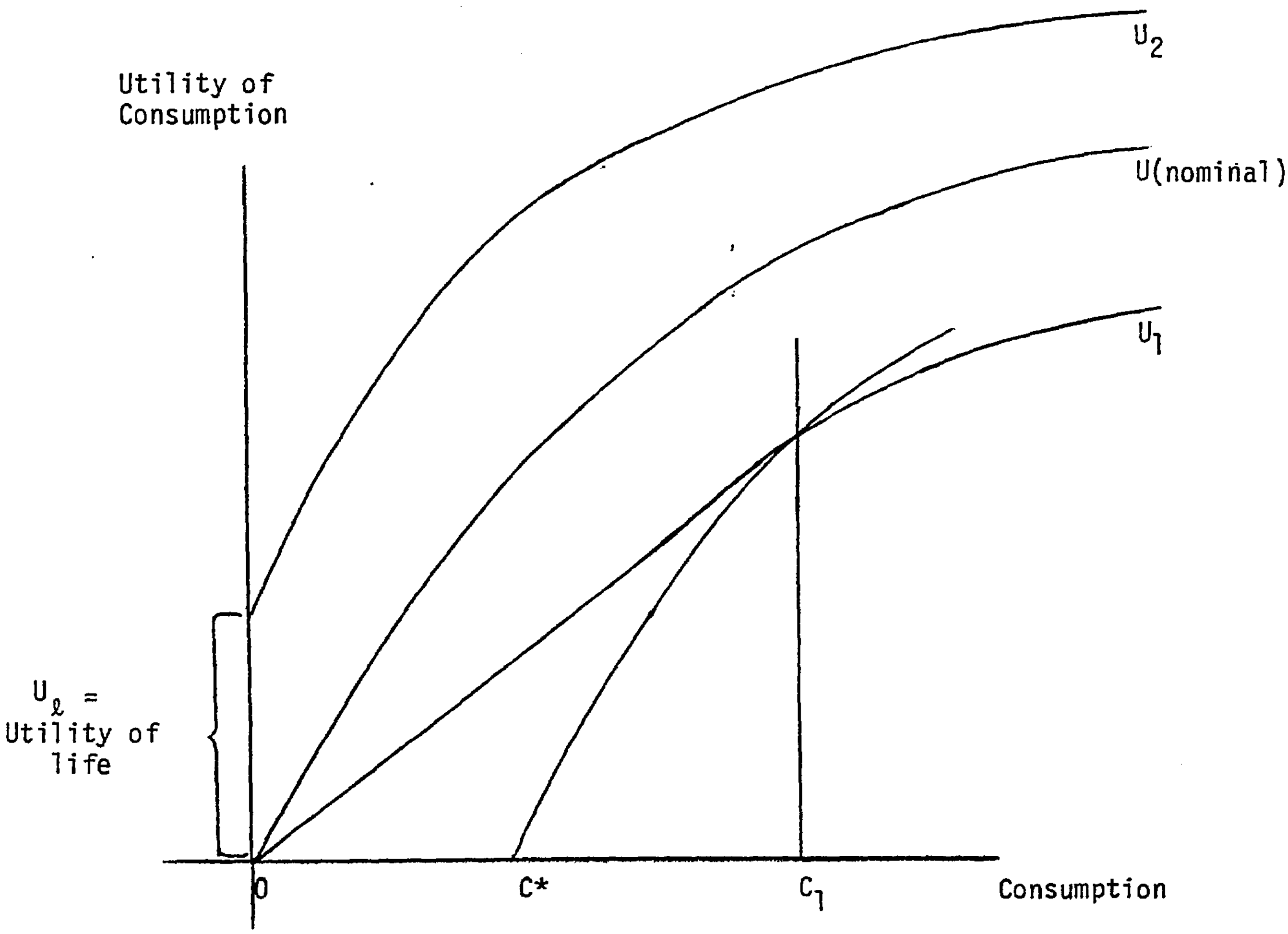
$$V_1 = \frac{C}{(1-p)\beta} \tag{A3}$$

In this case, value of life is directly proportional to consumption and inversely proportional to the survival probability and the elasticity of consumption.

Some Problems With This Simple Model

This model has recognized limitations, but it is important because it is often a starting point for more elaborate analysis. Fundamental to this model is the assumption that utility of living can be equated to the utility of consuming, and that the utility of death, C=0, is totally captured by the single point U(0) = 0. Even assuming this construct is valid, there are additional problems. Some authors have proposed that there exists a level of consumption C* > 0 such that when consumption falls below C*, life is viewed as being no better than death, i.e., U(C) = 0 for $0 \leq C < C^*$. Others have proposed that the property of being alive is valued in its own right and the utility of consumption is only part of the utility of being alive <u>and</u> consuming. These two alternatives are illustrated in figure 2. Note that the utility function U_1 implies there is some interval C* to C_1, where

FIGURE 2. TWO ALTERNATIVE UTILITY FUNCTIONS FOR THE SIMPLE MODEL COMPARED TO THE NOMINAL VERSION



C* = level of consumption equivalent to death

$$\frac{U_1(C)}{U_1'(C)} \leq C,$$

and therefore the value of life V_1, equation (A1), is less than C/(1-p), so that $V_1 \leq C$.[1] When consumption, C, is identified as the remaining income that a person will earn from that moment forward until death, (i.e., essentially human capital), this result has been interpreted as implying that the value of life as calculated by WTP is greater than the human capital approach if consumption is greater than some minimum value C_1, while less than the human capital approach for levels of consumption between C* and C_1.

If the utility function follows U_2 in figure 1, then life is valued separately and independently of consumption by the amount indicated, U_ℓ. When $U_2 = U_{nominal} + U_\ell$, value of life, V_1, is increased by the amount $U_\ell/(1\text{-}p)U_2'(C)$. Further V_1 will always be greater than C, assuming relation (A2) holds, and so the WTP approach will exceed the human capital approach. See Linnerooth (1979) for a more detailed analysis of these two cases.

This simple model can be generalized by considering more than one period, a bequest motive, and an allowance for insurance and annuities. These complications will be considered in turn in the following sections.

Annuities With No Bequest Motive

Assume that the basic model is modified so that the person has access to fair annuities and there is no bequest motive. Then it is clear that such a person would put all his wealth into an annuity that paid C/(1-p) if he survives the risk and 0 if he dies. Thus his expected utility is

$$E(V) = (1\text{-}p)\, U\left(\frac{C}{1\text{-}p}\right)$$

The total differential is

$$dE(V) = \left(- U\left(\frac{C}{1-p}\right) + (1-p)\, U'\left(\frac{C}{1-p}\right)\frac{C}{(1-p)^2}\right) dp$$

$$+ (1-p)\, U'\left(\frac{C}{1-p}\right)\left(\frac{1}{1-p}\right) dC$$

$$= 0.$$

Thus, value of life is given by the expression

$$V_2 = \frac{dC}{dp}$$

$$= \frac{U\left(\frac{C}{1-p}\right)}{U'\left(\frac{C}{1-p}\right)} - \frac{C}{1-p} \qquad \text{(A4)}$$

To show V_2 is an increasing function of C/(1-p), note

$$\frac{dV_2}{d\left(\frac{C}{1-p}\right)} = \frac{U'^2\left(\frac{C}{1-p}\right) - U\left(\frac{C}{1-p}\right) U''\left(\frac{C}{1-p}\right)}{U'^2\left(\frac{C}{1-p}\right)} - 1$$

$$= \frac{- U\left(\frac{C}{1-p}\right) U''\left(\frac{C}{1-p}\right)}{U'^2\left(\frac{C}{1-p}\right)}$$

and so the derivative is positive under the usual risk averse assumptions:

$$U''(C) < 0 < U'(C).$$

This shows that V_2 is an increasing function of C/(1-p). Thus, as in the simple model, V_2 increases with C and p. Interestingly it is not possible to determine whether access to annuities generally increases or decreases the value of life. For p close to zero, expression (A4) is approximately

$$\frac{U(C)}{U'(C)} - C$$

which would be less than $V_1 = U(C)/U'(C)$ when $p \to 0$, from (A1). However, as p increases, V_2 could become larger than V_1. For the particular case $U(C) = C^\beta$, $0 < \beta < 1$, the relationship is better determined:

$$V_2 = \left(\frac{C}{1-p}\right) \frac{1-\beta}{\beta} < V_1 \quad \text{(in equation A3)},$$

for all values of C, p and β. Thus access to annuities would decrease one's willingness to pay, presumably because some of the negative consequences of risk have been mitigated by increased consumption, should the individual live.

Bequest Motive: Multi-State Utility Functions

One of the more frequent generalizations to the simple model is to assume the utility function decomposes into two states such that the utility of wealth, W (as compared to consumption) has the form

$$U(W) = \begin{cases} L(W) \text{ if alive} \\ D(W) \text{ if dead.} \end{cases}$$

In particular, L(W) is the utility of being alive with wealth W, and D(W) is the utility of being dead and leaving an estate, W. This structure assumes the individual has no access to annuities or insurance, but allows an individual to have a bequest motive. To the extent that a bequest motive is a mitigating factor: one could expect, a priori, the value of life to decrease.

Again by taking the total differential of the expected utility

$$E(U(W)) = (1-p)L(W) + pD(W) \qquad (A5)$$

and solving for dW/dp, value of life is determined as

$$V_3 = \frac{L(W) - D(W)}{(1-p)L'(W) + pD'(W)}, \qquad (A6)$$

Jones-Lee (1974) showed that this expression is an increasing function of p and W under the normal risk averse assumptions and the assumption L(W) > D(W). This latter inequality implies living is preferred to death at all levels of wealth. Weinstein et al (1980) obtained similar results with somewhat weaker conditions. Thus this two state model exhibits properties similar to the simple model. Also it is clear that for D(W) > 0, $V_3 < V_1$ which validates the a priori expectation.

Smith and Gilbert (1984) suggest that more than two states may be required if people value a cancer death differently from a heart attack death or from an accident related death, etc. This generalization is only now being studied.[2]

The Effect of Fair Insurance and Annuities on the Two State Model

Insurance and annuities allow a person to allocate wealth between present and future consumption and the estate. Let W_L be wealth while alive and W_D be the size of the estate. Then

$$W_D - W_L$$

is the amount of the insurance (+) or annuity (-). With <u>fair</u> insurance and annuities, an appropriate budget constraint is

$$(1-p)\,W_L + p(W_D) = W. \tag{A7}$$

To see this, assume a person buys fair insurance having a premium of 1. So $W_L = W - 1$. With death, the insurance company pays 1/p and so $W_D = W - 1 + 1/p$. Substituting into expression (A7), the constraint is clearly satisfied. A similar argument applies to annuities.

The value of life can be calculated as before except now W_L and W_D are chosen to maximize (A5) subject to the above budget constraint. It is easily shown that this implies

$$L'(W_L) = D'(W_D) \ ,$$

and that

$$V_4 = W_D - W_L + \frac{L(W_L) - D(W_D)}{L'(W_L)} \tag{A8}$$

In particular, equation (A8) shows that the amount of insurance a person buys will usually understate the value of life. Cook and Graham (1977) show that under the normal risk averse assumption V_4 is an increasing function of p and W, and so behaves similar to the simple model. This result is in contrast to one obtained by Jones-Lee (1976) where he shows V_3, but with insurance, will not depend on p. However, Jones-Lee is careful to note that his conclusion is valid only in a small neighborhood of p, where an individual is not apt to adjust the level of insurance. For large variations in p, V will change and the Jones-Lee result is not applicable.

These four models are summarized in table 1. Broadly, they all exhibit similar characteristics and indicate that V increases with p and W (or C) and that these properties are fairly robust with regards to the assumptions invoked.

Multi-Period Model

The single period is useful for analyzing the value of life when the risk of death is immediate. But these models cannot determine the value of life as a function of age. Multi-period models, however, introduce several complicating factors. Such models need to reflect market interest rates and an individual's subjective time preference. Such models also need to include yearly consumption and saving patterns. A major problem is simply generalizing the single period utility function to a lifetime utility function. One solution has been to assume that expected lifetime utility has the form

$$E(U_L) = \sum_{i=1}^{T} q_i^* \quad D^{i-1}\, U(C_i) \qquad \text{(A9)}$$

where

D = 1/(1+d) is the subjective discount factor[3]

q_i^* = the probability of living through the end of year i

= $\prod_{j=1}^{i} q_j$ where

q_j = the probability of surviving the j^{th} year given one is alive at the beginning of the j^{th} year. It will be assumed that if death occurs, it occurs at the beginning of the year. It is also assumed $q_1 = 1$.

C_i = consumption in year i

$U(C_i)$ = the utility of consuming C_i in year i, and

T = maximum life span.

While this is an obvious generalization of the one period model, there has been little research as to its suitability. As Shepard and Zeckhauser (1982) remark, the use of

TABLE 1. SOME GENERIC SINGLE PERIOD MODELS FOR VALUE OF LIFE AS MEASURED BY WILLINGNESS TO PAY

	MODEL	TYPICAL ASSUMPTIONS	VALUE OF LIFE	PROPERTIES	REFERENCES
SINGLE STATE UTILITY FUNCTIONS	$E(U) = (1-p)\ U(C)$ where C = consumption $(1-p)$ = survival probability	$U''(C) < 0 < U'(C)$ $C = 0 \Rightarrow$ death $U(0) = 0$ No insurance or annuities. No bequest motive. p is exogenous.	$V_1 = \dfrac{U(C)}{(1-p)\ U'(C)}$	As a person's survival chances decrease, V increases at an increasing rate. Implicitly, this implies older people will pay more than younger people.	Freeman, Linnerooth, Violette & Chestnut
	$E(U) = (1-p)\ U\left(\dfrac{C}{1-p}\right)$	Same as above except there exist fair annuities. Thus a person buys an annuity that is worth $\dfrac{C}{1-p}$ if he survives, 0 if he is dead.	$V_2 = \dfrac{U\left(\frac{C}{1-p}\right)}{U'\left(\frac{C}{1-p}\right)} - \dfrac{C}{1-p}$ where $U' = \dfrac{\partial U}{\partial C}$	V is an increasing function of $\dfrac{C}{1-p}$ and therefore increases with p and C.	Cook, Conley
DOUBLE STATE UTILITY FUNCTIONS	$E(U) = (1-p)\ L(W) + p\ D(W)$ where $1-p$ = survival probability W = initial wealth $L()$ = utility of living $D()$ = utility of death	$L''(W) < 0 < L'(W)$ $D''(W) < 0 < D'(W)$ $D(W) < L(W)$ p is exogenous. No insurance or annuities. There is a bequest motive.	$V_3 = \dfrac{L(W) - D(W)}{(1-p)\ L'(W) + p\ D'(W)}$	V is an increasing function of wealth and a decreasing function of the original survival, 1-p.	Jones-Lee
	Same as above but subject to the budget constraint $(1-p)W_L + p\ W_D = W$ where W_L is wealth while alive and W_D is the value of the estate.	Same as above except there exist fair insurance and annuities.	$V_4 = W_D - W_L + \dfrac{L(W_L) - D(W_D)}{L'(W_L)}$ where $L'(W_L) = D'(W_D)$	$W_D - W_L$ is the value of the insurance (+) or annuity (-). $L(W_L) = D(W_D)$ only if fair insurance can fully compensate a person for his death. With fair insurance and annuities, V is increasing in p.	Cook & Graham

equation (A9) requires the heroic assumption that

> "an individual's utility over a life span of different lengths can be represented as a weighted sum of period utilities. By invoking this assumption, we join with most previous literature on lifetime consumption patterns."

An important exception to this approach is given in an article by Pliskin et al (1980) where lifetime utility is modeled as the utility of y additional years of life, at an average health state, q, with utility H(q). It has the form

$$U(y,q) = \begin{cases} \frac{1}{r}\left(\left(y\,H(q)\right)^{r} - 1\right) + r, & r \neq 0 \\ \ln y\,H(q), & r = 0 \end{cases}$$

where r is the risk aversion factor. However, the properties of this utility function are not well understood at this time.

Assuming utility can be specified as a weighted sum as in equation (A9), and assuming consumption and survival probabilities are exogenous, the value of life in year t, as evaluated in year i, would have the form (see Freeman 1979)

$$V_{it} = \frac{dC_i}{dq_t}$$

$$= \frac{\sum_{k=t}^{T} q_k^* D^{k-i} U(C_k)}{q_t\, q_i^*\, U'(C_i)}, \quad t \geq i \tag{A10}$$

As Freeman notes, there are no constraints on the resulting values of V_{it} and so various inconsistencies are possible. For example, one might expect V_{it} to equal $V_{i+1,t}$ except possibly due to a year's worth of interest. That is

$$V_{it} = R\, V_{i+1,t}, \tag{A11}$$

where $R = (1+r)^{-1}$, the discount factor for money. From equation (A10), this relationship would imply

$$U'(C_{i+1})\left(\frac{D}{R}\right)q_{i+1} = U'(C_i)$$

and more generally (assuming $q_i^* \doteq 1$)

$$U'(C_i)\left(\frac{D}{R}\right)^{i-1} q_i^* = U'(C_1) \qquad \text{(A12)}$$

When consumption is exogenous, there is no reason to expect equation (A12) above to apply. However, with the assumption that a person can optimize his life long consumption pattern, either through savings or through money markets, relationships similar to (A12) can be demonstrated.

Life Cycles Models

Some of the more recent analyses for valuing life (Cropper (1982), Arthur (1981), Shepard and Zeckhauser (1982)) have assumed a life cycle approach to consumption that lets an individual use annuities and life insurance to adjust consumption C_i in year i to maximize lifetime expected utility. This allows a person to borrow funds during low income years and save during peak earning periods. The account given here is due to Cropper (1984) and it differs only in detail with that developed by Shepard and Zeckhauser (1982), and Arthur (1981).

To begin, assume there is no bequest motive and that fair actuarial notes can be bought (annuities) and sold (regular loans guaranteed by life insurance). The assumption that there is no bequest motive implies the individual will convert all wealth into annuities. To prevent an individual from issuing an unlimited number of actuarial notes, Cropper (following Yaari (1965)) requires the number of notes outstanding in the last year, T, to be zero, where T is the maximum biological age limit. This can be shown to be equivalent to the condition

$$A + \sum_{i=1}^{T} (y_i - C_i) R^i q_i = 0 \tag{A13}$$

where

A = initial wealth and

y_i = income in year i.

To maximize expected lifetime utility, equation (A9), Cropper (as well as others) shows that the optimal consumption schedule, $\dot{C}_i$, subject to the above constraint requires

$$U'(C_i) = \left(\frac{R}{D}\right)^{i-1} U'(C_1) . \tag{A14}$$

Note that if R = D, then this equation implies optimal consumption, C_i, is constant for all i. Also note that except for the absence of the factor q_i^*, this equation is the same as equation (A12) determined earlier.

The value of life is shown by Cropper to be

$$V_{1t} = \frac{\sum_{i=t}^{T} q_i^* R^{i-1}\left(\frac{U(C_i)}{U'(C_i)} - C_i + y_i\right)}{q_t}. \tag{A15}$$

(Cropper actually obtains an expression that has the summation beginning at i = t + 1. This is because she assumes death will occur at the end of the year instead of at the beginning). It is also easy to show, given equation (A12), that

$$V_{1t} = R^{i-1} V_{it} \qquad i \leq t .$$

Thus the value of life at age t evaluated at various years prior to t, differs by the amount of interest that would accrue.

Under normal risk averse assumptions,

$$\frac{U(C_i)}{U'(C_i)} - C_i$$

is greater than zero and an increasing function of C_i. Thus V_{it} is composed of two positive terms, one of which is the present value of the remaining expected lifetime earnings (i.e., the human capital approach)

$$\sum_{i=t}^{T} q_i^* R^{i-1} y_i \quad ,$$

and the other a function of the utility.

Cropper also shows that Vit is an increasing function of $p_i = 1-q_i$ if and only if

$$\sum_{i=t}^{T} q_i^* R^{i-1} (C_i - y_i) \geq 0 \ . \qquad \text{(A16)}$$

With positive initial wealth, A>0, C_i will likely be greater than y_i, and so this condition will likely be satisfied. For those cases where condition (A16) does not hold, the multi-period model will behave differently from the single period model.

An Example: $U(C_i) = C_i^{\beta}$

To illustrate some of the properties of this lifecycle model assume the subjective discount factor D is equal to R (as in Shepard and Zeckhauser (1982), (1984)). Also assume initial wealth is zero. Then lifetime consumption is a constant over time and equal to

$$C_k = \bar{C}.$$

$$= \frac{\sum_{i=1}^{T} q_i^* \; R^{i-1} y_i}{\sum_{i=1}^{T} q_i^* \; R^{i-1}} \text{ , all } k$$

By making the additional assumption that $U(C_i) = C_i^{\beta}$, $0 < \beta < 1$, value of life can be expressed as

$$V_{1t} = \frac{\sum_{i=t}^{T} q_i^* \; R^{i-1} \left(\frac{1-\beta}{\beta} \bar{C} + y_i \right)}{q_t} \qquad \text{(A17)}$$

This result is essentially the same as that obtained by Shepard and Zeckhauser using a scenario they called the perfect market case, except they calculated the value of life conditional on being alive in year t. That is, they use

$$V_{tt}^* = V_{tt}/q_{t-1}^*.$$

Their analysis also considered an example they called the Robinson Crusoe case where an individual is assumed to have no access to actuarial money markets but is allowed to save and earn interest. By taking the earning profile (circa 1978) and life table of an average US male conditional on being alive at age twenty, they establish representative values of y_i and q_i^*, in equation (A17) above. The interest rate was set equal to 5%, and β was chosen as 0.20. This allowed them to estimate the value of life for various age brackets. Their results are summarized in table 2.

Broadly, the estimated maximum value of life falls in the 1 to 1.25 million dollar range and occurs at age 40 for the Robinson Crusoe case and at age 25 for the perfect market case. These estimates are inversely related with the assumed elasticity, β,

TABLE 2. VALUE OF LIFE AT VARIOUS AGES
FOR MALES WITH 1978 INCOME PROFILES

AGE	V^{*}_{tt} (1) ROBINSON CRUSOE CASE	PERFECT MARKET CASE	DISCOUNTED EXPECTED EARNINGS
20	0.50 million	1.05 million	0.20 million
40	1.25 million	0.97 million	0.22 million
50	1.00 million	0.80 million	0.16 million
60	0.63 million	0.60 million	0.09 million
70	0.33 million	0.42 million	0.00
80	0.10 million	0.29 million	

SOURCE: Shepard and Zeckhauser (1984)

[1] To reflect 1981 incomes, these values should be increased by about 28%. The * on V_{tt} indicated that this is conditional on being alive in year t, as developed by Shepard and Zeckhauser (1984).

however. Thus WTP would be larger or smaller than that given in table 2 depending upon whether an individual's own elasticity was smaller or larger than 0.20. For this reason, the result given in table 2 may be more useful for establishing how value of life might vary with age, than as actual estimates of the value of life. Indeed, Shepard and Zeckhauser also make this point. They also note that both scenarios represent extreme cases, and so would expect actual WTP to lie somewhere in between.

For many applications, value of life per se, is not as important as value of additional life year (or expected life years). These computations are given in table 3. For example, at age 20, a male can expect to live another 52 years. From table 2, the value of life (Robinson Crusoe case) at age 20 is 0.5 million. Thus each expected life year is worth 0.5/52 = $9,615.00. The Robinson Crusoe case indicates the value of each expected life year first increases and then decreases with age. For the perfect market case, the value of each expected life year increases with age, although this increase is often fairly modest. The increasing trend associated with the perfect market scenario implies that given a choice between two programs, each saving the same number of expected life years, but one directed at the elderly and the other directed at youth, the program for the elderly would be more highly valued. Note, however, that the equal expected life years requirement implies the program for the elderly must include more people. Note also that a different life table could give a very different pattern; and so the results here should not be generalized without further analysis.

Table 4 summarizes the multi-period models. Also indicated is a two state, multi-period model as developed by Cropper. The resulting expression for the value of life, however, was very complicated, even after making the simplifying assumption that utility has the form C_i^{β}. Thus no additional analysis is presented here.

Summary

This appendix has examined how theoretical models calculate value of life with

TABLE 3. VALUE OF LIFE YEARS AT VARIOUS AGES FOR US MALES

Age, t	Male Remaining Life Expectancy Assuming One Is Alive at Age t	Value of each expected life year conditional on living to age t		
		WTP Robinson Crusoe	WTP Perfect Market	Discounted Expected Earnings
20	52	9,600	20,200	3,800
40	33.9	36,900	28,600	6,500
50	25.3	39,500	31,600	6,300
60	18.2	34,600	32,000	4,950
70	12.4	26,600	34,100	0.0
80	7.7	13,000	37,700	0.0

TABLE 4. THREE MULTI-PERIOD VALUE OF LIFE MODELS

Model	Typical Assumptions	Value of Life	Properties	References
$E(U) = \sum_{i=1}^{T} q_i^* D^{i-1} U(C_i)$ where $U(C_i)$ is the utility of consuming C_i $D = \frac{1}{1+d}$ is the subjective discount rate q_i^* = probability of surviving for at least i years $= \prod_{j=1}^{i} q_j$ q_i = probability of surviving the ith year, given being alive at the beginning of the year T = maximum life span	$U''(C_i) < 0 < U'(C_i)$ C_i exogenous q_i^* exogenous No insurance or annuities No bequest motive	$V_{it} = \frac{\sum_{j=t}^{T} \frac{q_j^*}{q_t} D^{j-1} U(C_j)}{q_i^* \; U'(C_i)}$	$V_{it} = \frac{\partial C_i}{\delta q_t}$ is the value of life in year t as evaluation in year i. Consumption schedule not optimized. Thus, various inconsistencies are possible.	Freeman
$E(U) = \sum_{i=1}^{T} D^{i-1} q_i^* U(C_i)$ Same as above, and in addition there is the budget constraint. $A + \sum_{i=1}^{T} (y_i - C_i)R^{i-1} q_i^* = 0$	$U''(C_i) < 0 < U'(C_i)$ C_i endogenous q_i^* exogenous Fair annuities and insurance No bequest motive $A \geq 0$ initial wealth $y_i \geq 0$ earning in year i R = discount rate for money $= \frac{1}{1+r}$	$V_{it} = \frac{\sum_{j=t}^{T} q_j^* R^j \left[\frac{U(C_j)}{U'(C_j)} - C_j + y_j\right]}{R^i \quad q_t}$ where C_j are chosen so that $U'(C_j) = \frac{R}{D}^{i-1} U'(C_i)$	Value of life increases with probability of death provided $\sum_{i=t}^{T} q_i^* R^{i-1}(C_i - y_i) > 0.$ Willingness to pay exceeds expected life time earnings, the human capital approach.	Cropper, Arthur, Shepard and Zeckhauser
$E(U) = \sum_{i=1}^{T} D^{i-1} q_i^* U_L(C_i)$ $+ \sum_{i=1}^{T} p_i D^{i-1} U_D(S_i)$ where $U_L(C_i)$ is the utility of being alive in year i and consuming C_i. $D^i U_D(S_i)$ is the utility of being dead in year i leaving an estate S_i, and $P_i = \prod_{j=1}^{i-1} q_j \; (1-q_i).$	Same as above except there is a bequest motive. In addition, to obtain useful results it is necessary to assume $U_L(C_i) = C_i^B$ $U_D(S_i) = \delta S_i^B$ subject to the budget constraint $A + \sum_{i=1}^{T} (y_i - C_i - S_i) q^* r_i^i = 0.$	$V_{it} = \frac{\sum_{j=t}^{T} \left[\left(q_j^* + P_j \delta^{\frac{1}{1-B}} R^j\right) \frac{1-B}{B} C_j + q_j^* R^j y_j\right]}{R^i \quad q_t}$ $- q_t^* R^{t-1} \left(\frac{1-B}{B}\right) C_t$ where C_i are chosen so that $C_i = (R/D)^{\frac{i-1}{1-B}} C_1$ and $S_i = \delta^{\frac{1}{1-B}} C_i$		Cropper

regard to a number of factors:

- age,
- access to money markets,
- income,
- level of risk,
- degree of risk aversion,
- subjective and real discount rates, and
- bequest motive.

Table 5 summarizes these findings. Appendix B develops and illustrates the theory for calculating WTP for one life table vs another.

TABLE 5. SUMMARY RESULTS

Variable Factor	Effect on Value of Life
Fair Annuities and Insurance	For general utility function, fair annuities and insurance can either increase or decrease value of life, V. When $U = C_i^{\beta}$, access to annuities will decrease V. For multi-period models access to these financial instruments implies $$U'(C_t) = \left(\frac{R}{D}\right)^{t-1} U'(C_1)$$ Thus, C_t (and therefore $U(C_t)$) is increasing, constant or decreasing over time depending upon $$\left(\frac{R}{D}\right) \lesseqgtr 1.0$$ (and assuming U is an increasing concave function).
Bequest Motive	A bequest motive essentially requires a two-state utility function: L(W) and D(W). From equation (A6), value of life clearly decreases with a bequest motive. (This is clear by assigning D(W) = 0, all W.)
Level of Risk	For single period models, value of life increases with degree of risk p. This property was also found in the multi-period model provided $$\sum_i^T q_i^* R^i (C_i - y_i) > 0,$$ equation (A16).
Level of Consumption	Value of life increased with wealth/consumption for all models. This result has important implications for empirical studies since many of the hedonic wage models focused on blue collar jobs while surveys have questioned teachers/professionals.
Age	The multiperiod models show value of life does depend upon the age of the individual. The results are mixed, however, and depend upon what assumptions are made. Value of a life-year also changed, with age varying by a factor of two between highest and lowest value.
Utility Function	Most theoretical analyses assume the utility function is a concave increasing ($U'' < 0 < U'$) function of wealth or consumption. Multi-period models use a weighted sum of yearly utilities. Clearly more research is indicated here.

FOOTNOTES

1. Conley demonstrates that C_l could be infinite and therefore $V_l \leq C$ for all C.

2. A preliminary examination of the topic is found in an EPA report by Steve Beggs: Diverse Risks and the relative Worth of Government Health and Safety Programs: An Experimental Survey, June 1984.

3. For example Jones-Lee (1976) and Arthur (1981) do not discount utility over time (i.e., D=1.0). Keeler and Cretin discuss this issue with regards to cost benefit analysis when benefits are not monetary (such as the number of lives saved). They conclude that costs and benefits should be discounted at the same rate. However, their analysis does not address utility functions.

REFERENCES

1. Arthur, W.B. (1981). "The Economics of Risk to Life," American Economic Review, pp. 54-64.

2. Conley, B.L. (1976). "The Value of Human Life in the Demand for Safety," American Economic Review 66, pp. 45-55.

3. Cook, P.J. (1978). "The Value of Life in the Demand for Safety: Comment," American Review 68, pp. 710-711.

4. Cook, P.J. and D.A. Graham (1977). "The Demand for Insurance and Protection: The Case of Irreplaceable Commodities," Quarterly Journal of Economics, pp. 143-156.

5. Cropper, M.L. (1982). "Valuing Risks to Life," preliminary report.

6. Freeman, M. (1979). The Benefits of Environmental Improvement. Resources for the Future, Inc. Johns Hopkins University Press.

7. Jones-Lee, M.W. (1974). "The Value of Changes in the Probability of Death or Injury," Journal of Political Economy 82, pp. 835-849.

8. Jones-Lee, M.W. (1976). The Value of Life: An Economic Analysis, University Chicago Press, Chicago.

9. Keeler, E.B. and S. Cretin (1983). "Discounting of Life-Saving and other Nonmonetary Effects." Management Science, March 1983, pp. 300-306.

10. Linnerooth, J. (1979). "The Value of Human Life: A Review of the Models," Economic Inquiry Vol. XVII, pp. 52-74.

11. Pliskin, J.S., D.S. Shepard and M.C. Weinstein (1980). "Utility Functions for Life Years and Health Status," Operations Research 28, pp. 206-224.

12. Shepard, D.S. and R.J. Zeckhauser (1984). "Survival Versus Consumption," Management Science Vol. 30, pp. 423-439.

13. Shepard, D.S. and R.J. Zeckhauser (1982). "Life Cycle Consumption and Willingness to Pay for Increased Survival." The Value of Life and Safety, M.W. Jones-Lee ed. North-Holland Publishing Company.

14. Smith, V.K. and C.S. Gilbert (1984). "The Valuation of Environmental Risk Using Hedonic Wage Models," preliminary report.

15. Thaler, R. and S. Rosen (1975). "The Value of Life Saving," Household Production and Consumption, N.E. Terleckyj ed. Columbia University Press, New York,

16. Violette, D.M. and L.G. Chestnut (1983). Valuing Reductions in Risk:' A Review of the Empirical Estimate.

17. Weinstein, M.C., D.S. Shepard and J.S. Pliskin (1980). "The Economic Value of Changing Mortality Probabilities: A Decision-Theoretical Approach," Quarterly Journal of Economics 94, pp. 375-396.

18. Yaari, M.E. (1965). "Uncertain Lifetime, Life Insurance and the Theory of the Consumer," Review of Economic Studies, pp. 137-150.

APPENDIX B: THE CALCULATION OF COMPENSATING AND EQUIVALENT SURPLUS FOR TWO LIFE TABLES, THE PERFECT MARKET CASE

Value of a life, as calculated by willingness to pay, is usually defined as the aggregate amount of money a large group of people would pay to reduce the number of deaths among them by one. For example, if a group of 100,000 people were willing to pay, on average, $10.00 each to reduce the number of deaths by one (i.e., a reduction in the death rate by 0.00001), then life would be valued as 100,000 x $10.00 or one million dollars. In actual practice, most life savings programs adjust several years of a person's life table. This complication has not been adequately addressed in the literature, and in particular, it is of interest to determine what a person's willingness to pay would be for one life table in place of another.

Valuing a Life Table

Assume two life tables are available, $q_i^*(o)$ and $q_i^*(n)$, where $q_i^*(o)$, $t=$ 1,...T, are the original survival probabilities, and $q_i^*(n)$s are new probabilities that reflect some health improvement. Then following the life cycle theory of Cropper (1982) and Shepard and Zeckhauser (1984) as described in appendix A, the expected utility under the original life table would be

$$E(U_o) = \sum_{i=1}^{T} q_i^*(o) \quad D^{i-1}U(C_i(o))$$

where D = $(1+d)^{-1}$ is the subjective discount factor, and $C_i(o)$ is the original consumption pattern, This expected utility would change (presumably increase) to

$$E(U_n) = \sum_{i=1}^{T} q_i^*(n) \quad D^{i-1}U(C_i(n))$$

for the new life table.

Under the perfect market scenario, (i.e., annuities and insurance) Cropper (1982) showed that consumption in year t would be constant and would equal (assuming no initial wealth)

$$\bar{C} = \left(\sum_{i=1}^{T} q_i^* \; R^{i-1} y_i \right) \Big/ \left(\sum_{i=1}^{T} q_i^* \; R^{i-1} \right), \qquad \text{(B1)}$$

where R (assumed equal to D)[1] is the discount factor, for money, and y_i is the income in year i. Thus expected utility for the original life table simplifies to

$$E(U_o) = U(\bar{C}_o) \sum_{i=1}^{T} q_i^*(o) \; D^{i-1}$$

$$= U(\bar{C}_o) \cdot DLY_o \qquad \text{(B2)}$$

where

$$DLY_o = \sum_{i=1}^{T} q_i^*(o) \; D^{i-1} \qquad \text{(B3)}$$

is the expected discounted life years (in physical units) associated with the original life table and annual consumption is, by equation (B1),

$$\bar{C}_o = \left(\sum_{i=1}^{T} q_i^*(o) \; R^{i-1} y_i \right) \Big/ DLY_o \; . \qquad \text{(B4)}$$

Similar formulas for $E(U_n)$ and $\bar{C}_n$ for the new schedule are easily derived. Note that the expected utility has been simplified to only a function of yearly consumption, $\bar{C}$, and discounted life years. This will be the basis for valuing one life table over another.

The basic question is what would a person pay to get or keep a new and improved life table when compared to the original table (sometimes called compensating surplus), or what amount of money would an individual demand to forego the new life table (equivalent surplus).[2] Figure 1 illustrates these two calculations. The points A and B in figure 1 are determined by the two life tables, $q_t^*(o)$ and $q_t^*(n)$, and the equations (B2) and (B3). The distance B to D is the compensating surplus (CS), and represents the maximum annual amount of money a person with the new life table could pay and still be as well off (in terms of utility) as with the old life table. The distance A to E is the equivalent surplus (ES) and would be the amount of additional money a person with the old life table would require to be as well off as he would be with the new life table at B.

To calculate compensating surplus note that equation (B2) implies $\bar{C}_3$, in figure 1, is equal to

$$\bar{C}_3 = U^{-1} \frac{E(U_o)}{DLY_n} \quad ,$$

where U^{-1} is the inverse function. Thus compensating surplus is

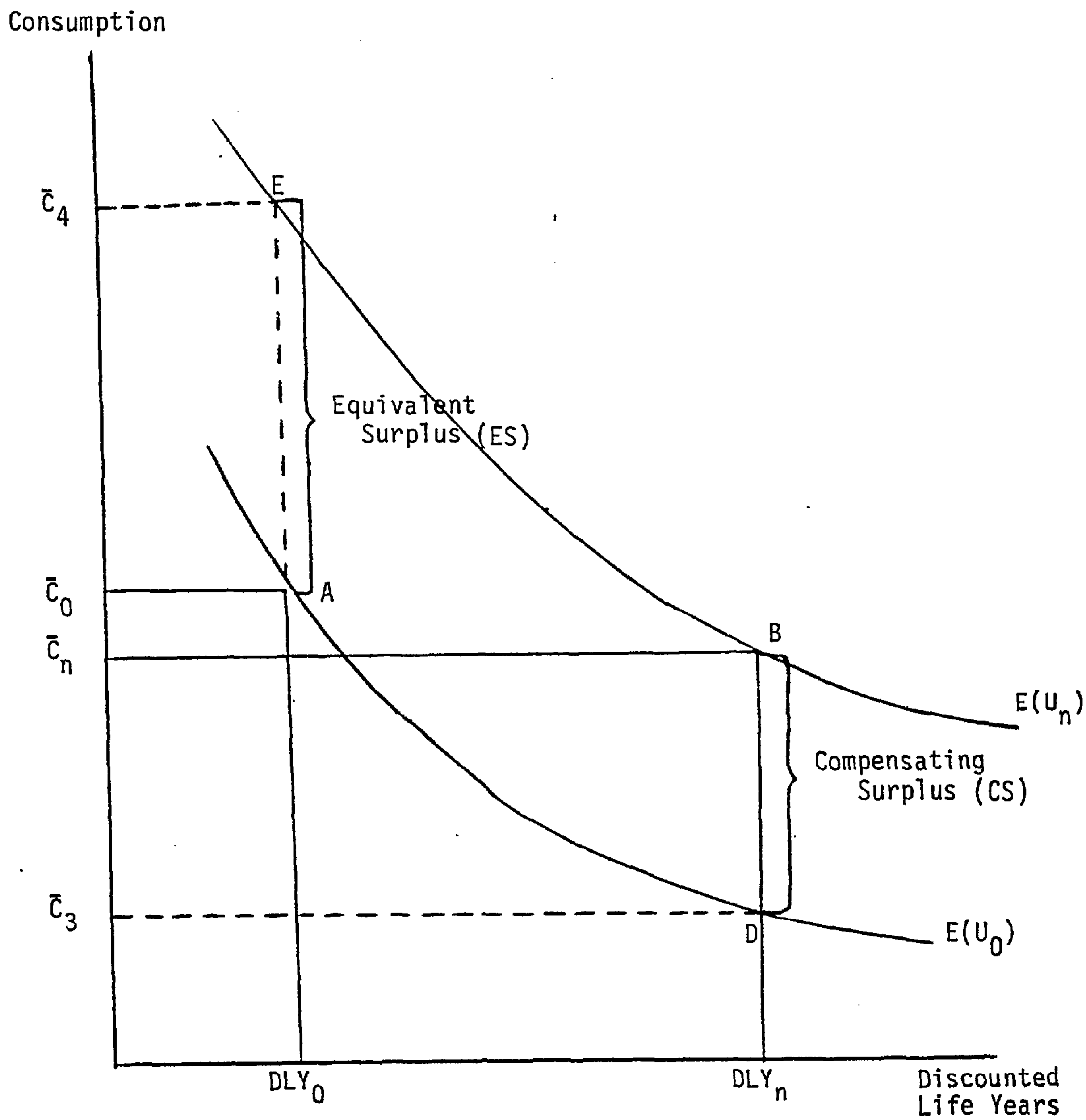
$$\begin{aligned} CS &= \bar{C}_n - \bar{C}_3 \\ &= \bar{C}_n - U^{-1} \frac{E(U_o)}{DLY_n} \quad . \end{aligned} \qquad \text{(B5)}$$

This is the maximum amount a person with the new life table could forego on an annual basis and still be as well off with the old life table.

The expected utility in year t, conditional on being alive in year t, is

$$\begin{aligned} E(U_o \mid t) &= U(\bar{C}_o) \cdot \left(\frac{\sum_{i=t}^{T} R^i \; q_i^*(o)}{R^t \; q_t^*(o)} \right) \\ &= U(\bar{C}_o) \; DLY_t(o) \end{aligned}$$

FIGURE 1. THE CALCULATION OF COMPENSATING
AND EQUIVALENT SURPLUS



$$DLY = \sum_{i=1}^{T} D^{i-1} q_i^*$$

$$E(U) = U(\bar{C}) \cdot DLY$$

where

$$DLY_t(o) = \frac{\sum_{i=t}^{T} R^{i-t} \, q_i^*(o)}{q_t^*(o)} \tag{B6}$$

is the expected discounted life years remaining conditional on being alive in year t. Thus compensating surplus in year t, conditional on being alive, would be

$$CS_t = \bar{C}_n - U^{-1}\left(\frac{U(\bar{C}_o) \; DLY_t(o)}{DLY_t(n)}\right) \tag{B7}$$

where $DLY_t(n)$ is calculated as in equation (B6) using $q_i^*(n)$. A similar analysis shows that the equivalent surplus is given by the equation

$$ES_t = U^{-1}\left(\frac{U(\bar{C}_n) \; DLY_t(n)}{DLY_t(o)}\right) - \bar{C}_o \quad . \tag{B8}$$

The values CS_t and ES_t represent the maximum amount of money an individual would pay or the minimum amount he would demand <u>each year</u> to make him indifferent to the two life tables, conditional on being alive in year t. The total expected compensating surplus, summed over a person's remaining life time and discounted back to year t, would be

$$TCS_t = CS_t \cdot \left(\frac{\sum_{i=t}^{T} q_i^*(n) \; R^{i-t}}{q_t^*(n)}\right)$$

$$= CS_t \; DLY_t(n) \quad . \tag{B9}$$

Similarly, the total discounted equivalent surplus would be

$$TES_t = ES_t \cdot DLY_t(o) \quad . \tag{B10}$$

These two quantities give alternative measures of the total value of the new life table over the old one, assuming the individual was alive in year t.

An Example

To demonstrate these formulas, the example of Shepard and Zeckhauser (1982) will be used. Relevant quantities are given in table 1. (Some of these numbers are only approximate, since they were estimated off of graphs presented in that paper. The number of significant digits reported in table 1 exceed that which would be required. It is intended to permit the reader to make independent verification of the results. Actual estimates of the value of a one life table compared to another have large uncertainties, and rounded values would be appropriate in other circumstances.) To simplify the calculations, Shepard and Zeckhauser divided age into five year intervals. The survival probability $q_i^*(o)$, and income data, y_i, are assumed to apply to each year in that interval. Maximum income is taken to be $24,000 at age 50 and this corresponds approximately to the median amount earned in 1981 by a male worker in the 45 to 55 age bracket, according to the Bureau of Census. The value of y_i in table 1 is the ratio of income in year i to this maximum amount. A discounted life interval conditional on being alive in year t, $DLY_t(o)$, is calculated as in formula (B6), and these numbers should be multiplied by 5 to calculate discounted life years. The value of a statistical life, V_{tt}, conditional on being alive in year t, (also see appendix A) is calculated by Shepard and Zeckhauser:

$$V_{tt} = \frac{\sum_{i=t}^{T} q_i^*(o)\ R^{i-t} \left(\frac{U(\bar{C})}{U'(\bar{C})} - \bar{C} + y_i \right)}{q_t^*} .$$

TABLE 1. THE EXAMPLE OF SHEPARD AND ZECKHAUSER
(updated to 1981 dollar)

i	Age(1)	Income(2) y_i	Survival Probability $q^*_i(o)$	Discount(3) Factor $R^{5(i-1)}$	$DLY_i(o)$(4)	$DE_i(o)$(5)	Value of Life V_{tt}(6) (millions)
1	20	0.28	1.000	1.000	4.1255	2.7060	1.6238
2	25	0.59	0.991	0.784	4.0253	3.1224	1.6420
3	30	0.79	0.982	0.614	3.8965	3.2637	1.6152
4	35	0.88	0.971	0.481	3.7386	3.1935	1.5593
5	40	0.95	0.957	0.377	3.5463	2.9949	1.4759
6	45	0.99	0.935	0.295	3.3263	2.6753	1.3683
7	50	1.00	0.900	0.231	3.0845	2.2348	1.2393
8	55	0.99	0.848	0.181	2.8235	1.6702	1.0894
9	60	0.95	0.772	0.142	2.5564	0.9500	0.9189
10	65	0	0.670	0.111	2.2888	0	0.7206
11	70	0	0.541	0.087	2.0371	0	0.6413
12	75	0	0.402	0.680	1.7812	0	0.5608
13	80	0	0.263	0.590	1.5241	0	0.4799
14	85	0	0.138	0.042	1.2670	0	0.3982
15	90	0	0.0410	0.033	1.1462	0	0.3608
16	95	0	0.0071	0.026	1.0772	0	0.3470
17	100	0	0.0007	0.020	1.0000	0	0.3148

(1) Life table is conditional on having lived to age twenty.
(2) Income relative to maximum, that is assumed to occur at age 50. Shepard and Zeckhauser set this maximum to $18,000 (circa 1978). This had increased to $24,000 by 1981.
(3) Discount factor R = 1/ 1.05. The factor of 5 is due to the use of 5 year intervals.
(4) $DLY_i(o)$ is discounted life intervals conditional on being alive in year i, as in equation (B6). These values should be multiplied by 5 to calculate discounted life years.
(5) $DE_i(o)$ discounted earning conditional on being alive in year i. These values should be multiplied by 5 and then by $24,000.
(6) As calculated in equation (11). Even after correcting for the change from $18,000 to $24,000, these values differ from Shepard and Zeckhauser due to slightly different treatment of the discount rate and because the life table has been extended from 90 to 100 years.

When the utility function has the form C^{β} , value of life simplifies to

$$V_{tt} = \frac{\bar{C}\ (1-\beta)\ DLY_t(o)}{\beta} + DE_t(o) \qquad \text{(B11)}$$

where $DE_t(o)$ is expected discounted life earnings conditional on being alive in year t,

$$DE_t(o) = \sum_{i=t}^{T} q_i^*(o)\ R^{i-t}\ y_i \quad .$$

When $t=1$ equation (B11) simplifies even further to

$$V_{tt} = \frac{\bar{C}\ DLY_t(o)}{\beta} \quad .$$

Shepard and Zeckhauser chose β to be equal to 0.20. From equation (B4), annual consumption, $\bar{C}_o$, is calculated to be \$15,744.00.[3] To illustrate equation (B11), assume age t is 20. Then from table 1, the number of discounted life years remaining is

$$\begin{aligned} DLY_{20}(o) &= 4.1255 \times 5 \\ &= 20.6275, \end{aligned}$$

where the factor 5 is due to the use of 5 year intervals. Similarly, discounted earning is calculated as

$$\begin{aligned} DE_{20} &= 2.7060 \times 24{,}000 \times 5 \\ &= \$324{,}720.00. \end{aligned}$$

Thus, value of life at age 20 is

$$\begin{aligned} V_{20,20} &= (\$15{,}744)\ (20.6275)\ (0.8/0.2) + \$324{,}720.00 \\ &= \$1{,}623{,}757.00. \end{aligned}$$

TABLE 2. A LIFE TABLE FOR MALES THAT ASSUMES CARDIOVASCULAR DISEASES ARE ELIMINATED

i	Age	$q_i^*(n)^{(1)}$	$DLY_i(n)$	$DE_i(n)$	Value of Life $V_{tt}^{(2)}$ (millions)
1	20	1.000	4.2643	2.7447	1.6468
2	25	0.991	4.2040	3.1727	1.6796
3	30	0.983	4.1225	3.3246	1.6727
4	35	0.974	4.0220	3.2655	1.6345
5	40	0.964	3.8969	3.0750	1.5730
6	45	0.950	3.7518	2.7563	1.4899
7	50	0.932	3.5798	2.2981	1.3818
8	55	0.904	3.3946	1.7057	1.2535
9	60	0.866	3.1903	0.9500	1.0997
10	65	0.814	2.9740	0	0.9189
11	70	0.743	2.7601	0	0.8527
12	75	0.665	2.5098	0	0.7754
13	80	0.573	2.2364	0	0.6910
14	85	0.469	1.9278	0	0.5956
15	90	0.325	1.7089	0	0.5280
16	95	0.200	1.4701	0	0.4542
17	100	0.120	1.0000	0	0.3090

(1) The increase in survival probability assuming all cardio-vascular diseases are eliminated is as calculated by S.H. Preston (1972) and (1976) and are based on 1964 US life tables.

(2) Calculated as in table 1 assuming maximum earnings are $24,000 and R = $(1.05)^{-1}$.

Table 2 gives a new life table that assumes all cardiovascular diseases are eliminated. These values were developed by S.H. Preston et al (1972) and (1976). Table 3 shows that this new life table prolongs life on average 10.3 years at age 20. The chance of living to age 100 has been increased by nearly a factor of 30. Because cardiovascular diseases usually occur late In life when a person is no longer earning an income, annual consumption under the new life table is about $300 less per year than that associated with the old table.

Under the assumption that the utility function has the form $U(C) = C^{\beta}$, the formulas for compensating and equivalent surplus, equations (B7) and (B8), simplify to

$$CS_t = \bar{C}_n - \bar{C}_o \left(\frac{DLY_t(o)}{DLY_t(n)} \right)^{1/\beta} \qquad \text{(B12)}$$

and

$$ES_t = \bar{C}_n \left(\frac{DLY_t(n)}{DLY_t(o)} \right)^{1/\beta} - \bar{C}_o \quad . \qquad \text{(B13)}$$

For example, at age 20, compensating surplus, on an annual basis would be

$$\begin{aligned} CS_{20} &= 15,448 - 15,744 \left(\frac{4.1255}{4.2643} \right)^{\frac{1}{.2}} \\ &= 15,448 - 13,343 \\ &= \$2,105.00 \end{aligned}$$

From equation (B9), the total expected net present value would be

$$\begin{aligned} TCS_{20} &= 2,105 \times (4.2643 \times 5) \\ &\doteq \$44,882.00 \quad . \end{aligned}$$

The results for other years are summarized in table 4. The compensating surplus

TABLE 3. A COMPARISON OF THE ORIGINAL LIFE TABLE, $q_t(o)$, AND THE NEW LIFE TABLE ASSOCIATED WITH THE ELIMINATION OF CARDIOVASCULAR DISEASES, $q_t(n)$

		Original Life Table	New Life Table	Gain (Years)
Expected life years at age	20	72.1	82.4	10.3
	40	73.9	84.2	10.3
	60	78.4	87.6	9.2
	80	88.6	94.7	6.1
Annual Consumption	$\bar{C}$	$15,744	$15,448	

TABLE 4. COMPENSATING AND EQUIVALENT SURPLUS (MALE) FOR A LIFE TABLE WITHOUT CARDIOVASCULAR DISEASE

Age	Compensating Surplus: The maximum amount a person would pay to keep the new life table, conditional on being alive at age t.		Equivalent Surplus: The minimum amount a person would require to forfeit the new life table, conditional on being alive at age t.	
	Yearly Annuity Payment	Expected Net Present Value	Yearly Annuity Payment	Expected Net Present Value
20	2,105	44,882	2,486	51,264
25	2,779	58,412	3,453	69,496
30	3,573	73,645	4,736	92,275
35	4,523	90,965	6,519	121,845
40	5,622	109,552	9,008	159,734
45	6,824	128,021	12,458	207,201
50	7,971	142,679	16,784	258,855
55	9,181	155,825	23,061	325,566
60	10,247	163,458	31,018	396,476
65	11,198	166,509	41,475	474,645
70	12,000	165,609	54,796	558,128
75	12,614	158,286	70,059	623,950
80	13,133	146,859	89,344	680,842
85	13,529	130,403	110,983	702,244
90	13,287	113,526	96,776	555,884
95	11,551	84,908	46,674	259,463
100	-294	-1,471	-294	-1,471

Assumptions: $\beta = 0.20$
$R = (1.05)^{-1}$
Maximum earnings = $24,000 (1981)

associated with the new life table has a maximum value of $166,509 at age 65 with payments of $11,198 per year. The annual payment schedule continues to increase until age 85 when a person would be willing to forego nearly 90% of his income ($13,529) to keep the new life table. A negative value is actually recorded at age 100. This is because both life tables are identical at this point--that is, everyone who survives to age 100 will live to age 105 and then die--but because annual consumption is slightly larger under the old life table, it would now be preferred. The values calculated for compensating surplus in table 4 are disturbingly large. Yet cardiovascular disease is by far the greatest cause of death in the United State's (nearly 60% of all male deaths). A life table without this disease would certainly be highly valued, particularly by people with a strong risk aversion, as assumed here.

Equivalent surplus exceeds compensating surplus at all age levels and indicates that at age 85, a person would require $702,000.00 to give up the new life table, and the associated annual annuity payments would be over one hundred thousand dollars. As Arthur points out in his analysis, (one that also differs from that presented here, as discussed later) the large values associated with this new life table do

> "not imply the United States should spend corresponding amounts per person on the elimination of cardiovascular diseases. A flood of research dollars would by no means guarantee such a breakthrough. The illustration, however, gives an idea of the potential return to the individual."

In actual fact most health and environmental programs would change a life table only a very small amount (e.g., on the order of one death in a hundred thousand). Such a small change would add only a few minutes or hours to an average life time. To understand the implications of a small change in probability, assume a medical procedure is available that would immunize one person in 100,000 from a fatal cardiovascular disease (e.g., one person in 100,000 gets the new life table given in table 2). The survival probability for the entire population would be a weighted average and would be calculated as

$$q_i^*(n_l) \doteq q_i^*(o) + \frac{(q_i^*(n) - q_i^*(o))}{100{,}000} \tag{B14}$$

where $q_i^*(o)$ is the original survival probability, table 1, and $q_i^*(n)$ is the life table without cardiovascular disease as given in table 2. The difference between $q_i^*(n_l)$ and $q_i^*(o)$ is very small and for all practical purposes one can assume yearly consumption would not differ (i.e., $\bar{C}_o = \bar{C}_{n_l}$). Further, compensating surplus will be nearly equal to equivalent surplus.

Using the approximation,

$$(1 + x)^{\beta} = 1 + \beta x,$$

valid for small x, compensating surplus is approximated by the expression

$$CS_t = \frac{\bar{C}_o \; (DLY_t(n_l) - DLY_t(o))}{DLY_t(o)\,\beta}$$

$$= \frac{\bar{C}_o \; (DLY_t(n) - DLY_t(o))}{100{,}000\; DLY_t(o)\;\beta}$$

and the total net present value for a life table with survival probabilities, $q_i^*(n)$, would be

$$TCS_t \doteq \frac{\bar{C}_o \; (DLY_t(n_l) - DLY_t(o))}{\beta} \tag{B15}$$

$$\doteq \frac{\bar{C}_o \; (DLY_t(n) - DLY_t(o))}{100{,}00\;\beta} \quad .$$

Table 5 shows various values of CS_t and TCS_t and indicates that a procedure that could immunize one person in every 100,000 against cardiovascular diseases would be valued at 54¢ per person at age 20. This increases to a maximum of $2.87 at age 75.

These values are not very large per person, but in aggregate these could amount to

TABLE 5. COMPENSATING SURPLUS PER MALE (ALSO EQUAL TO EQUIVALENT SURPLUS) FOR A LIFE TABLE THAT ASSUMES 1 MALE IN 100,000 CAN BE IMMUNIZED AGAINST CARDIOVASCULAR DISEASE

Age	Compensating Surplus	
	Yearly Annuity Payment (dollars)	Net Present Value (dollars)
20	0.0265	0.54
25	0.0350	0.70
30	0.0456	0.89
35	0.0597	1.12
40	0.0778	1.38
45	0.1007	1.67
50	0.1264	1.95
55	0.1592	2.24
60	0.1951	2.49
65	0.2356	2.69
70	0.2793	2.85
75	0.3219	2.87
80	0.3678	2.81
85	0.4105	2.66
90	0.3864	2.21
95	0.2871	1.54
100	0	0

Assumptions: $\beta = 0.20$
$R = (1.05)^{-1}$
Maximum earnings = \$24,000 (1981)

substantial sums if enough people were affected. For example, if a health program could immunize 1 male in 100,000 against cardiovascular disease for the approximately 75 million males over 20 years of age in the United States, and assuming they were distributed by age as given in table 18, then the total compensating surplus would be

$$75 \text{ million} \left(\sum_i (\text{percent population})_i (\text{compensating surplus})_i \right)$$

$$= 75 \text{ million } ((0.0671) \times (0.54) + (0.0579) \times (0.70) + \ldots)$$

$$= 75 \text{ million } (0.9123)$$

$$= 68.4 \text{ million dollars.}$$

A similar calculation for females would essentially double the amount, as shown in a later section.

Other Causes of Death

Cardiovascular disease is certainly the major cause of male deaths. Table 6 indicates the age patterns for this and two other causes of death in males: neoplasms and motor vehicle fatalities. Tables 7 and 8 give the corresponding life tables that assume these causes are eliminated. The compensating surplus for a life without neoplasms is shown in table 9 and is calculated to be nearly $15,000 at age 20, increasing to nearly $49,000 at age 60. Thus this table is valued at approximately 1/3 of that associated with the elimination of cardiovascular diseases, as given in table 4. Also the life table without neoplasms is valued less at older ages. For example, by age 90, the compensating surplus associated with neoplasms is $6,269.00 or 13% of the maximum that is achieved at age 60. In contrast compensating surplus at age 90 associated with cardiovascular disease is still 68% of its maximum.

The age distribution for motor vehicle fatalities in table 6 is substantially different

TABLE 6. AGE PATTERNS OF INCIDENCE FOR THREE CAUSES OF DEATH AMONG MALES (CONDITIONAL ON LIVING AT AGE 20)

Age	Cardiovascular Disease	Neoplasm	Motor Vehicles
20	0.07%	0.35%	17.10%
25	0.13%	0.49%	11.15%
30	0.29%	0.70%	8.73%
35	0.69%	1.11%	7.56%
40	1.48%	2.04%	6.98%
45	2.69%	3.57%	6.89%
50	4.51%	6.34%	6.94%
55	6.95%	9.73%	6.71%
60	9.55%	13.28%	6.22%
65	12.75%	16.16%	6.31%
70	14.50%	15.79%	5.64%
75	15.17%	13.57%	4.66%
80	14.34%	9.50%	3.36%
85+	16.87%	7.36%	1.75%
Fraction of all Deaths (age 20)	59.37%	16.22%	2.33%

Reference S.H. Preston (1972) and (1976).

TABLE 7. A LIFE TABLE FOR MALES THAT ASSUMES ALL NEOPLASMS ARE ELIMINATED

i	Age	$q_i^*(o)^{(1)}$	$DLY_i(n)$	$DE_i(n)$	Value of Life $V_{\dagger\dagger}$ (millions)
1	20	1,000	4.1671	2.7231	1.6339
2	25	0.991	4.0788	3.1445	1.6567
3	30	0.983	3.9614	3.2887	1.6372
4	35	0.974	3.8145	3.2193	1.5829
5	40	0.961	3.6407	3.0249	1.5259
6	45	0.942	3.4383	2.7057	1.4032
7	50	0.914	3.2073	2.2570	1.2769
8	55	0.870	2.9596	1.6830	1.1303
9	60	0.807	2.69 63	0.9500	0.9598
10	65	0.723	2.4164	0	0.7580
11	70	0.609	2.1461	0	0.6731
12	75	0.478	1.8637	0	0.5846
13	80	0.334	1.5776	0	0.4948
14	85	0.191	1.2991	0	0.4075
15	90	0.0618	1.1635	0	0.3649
16	95	0.0118	1.0920	0	0.3429
17	100	0.0014	1.0000	0	0.3137

(1) As calculated by S.H. Preston (1972) and (1976).

Assumptions: $\beta = 0.20$
$R = (1.05)^{-1}$
Maximum earnings = $24,000 (1981)

TABLE 8. A LIFE TABLE FOR MALES THAT ASSUMES ALL MOTOR VEHICLE FATALITIES ARE ELIMINATED

i	Age	$q_i^*(o)^{(1)}$	$DLY_i(n)$	$DE_i(n)$	Value of Life V_{tt} (millions)
1	20	1.0	4.1547	2.7284	1.6370
2	25	0.995	4.0466	3.1387	1.6522
3	30	0.988	3.9158	3.2778	1.6277
4	35	0.980	3.7518	3.2018	1.5669
5	40	0.967	3.5593	3.0020	1.4822
6	45	0.946	3.3389	2.6812	1.3743
7	50	0.913	3.0930	2.2366	1.2434
8	55	0.861	2.8325	1.6712	1.0934
9	60	0.7854	2.5653	0.9500	0.9226
10	65	0.6837	2.2927	0	0.7227
11	70	0.5534	2.0414	0	0.6428
12	75	0.4119	1.7846	0	0.5619
13	80	0.2704	1.5262	0	0.4806
14	85	0.1433	1.2673	0	0.3990
15	90	0.0426	1.1465	0	0.3610
16	95	0.0074	1.0773	0	0.3392
17	100	0.00073	1.0000	0	0.3149

(1) As calculated by S.H. Preston (1972) and (1976).

Assumptions: $\beta = 0.20$
$R = (1.05)^{-1}$
Maximum earnings = $24,000 (1981)

TABLE 9, COMPENSATING AND EQUIVALENT SURPLUS FOR A LIFE TABLE WITHOUT NEOPLASMS: MALES

Age	COMPENSATING SURPLUS		EQUIVALENT SURPLUS	
	Yearly Annuity Payment	Expected Net Present Value	Yearly Annuity Payment	Expected Net Present Value
20	712	14,833	748	15,440
25	949	19,346	1,134	22,825
30	1,191	23,593	1,294	25,206
35	1,448	27,619	1,602	29,933
40	1,880	34,223	2,144	38,016
45	2,345	40,302	2,766	46,012
50	2,734	43,845	3,323	51,258
55	3,244	48,001	4,105	57,949
60	3,623	48,838	4,728	60,435
65	3,682	44,492	4,830	55,276
70	3,556	38,155	4,615	47,008
75	3,129	29,150	3,923	34,943
80	2,435	19,267	2,895	22,058
85	1,907	12,393	2,180	13,785
90	1,078	6,269	1,161	6,657
95	980	5,350	1,049	5,650
100	-58	-291	-58	-291

Assumptions: $\beta = 0.20$
$R = (1.05)^{-1}$
Maximum earnings = $24,000 (1981)

from that associated with either cardiovascular fatalities or fatal neoplasms. The highest risk occurs at age 20 and then declines fairly consistently. Thus, the impact of motor vehicle fatalities is largest during the period when a person has earned income, and as might be expected, consumption $\bar{C}$ for this life table actually increases slightly from $15,744 to $15,761. Table 10 shows compensating surplus and equivalent surplus are both at a maximum at age 20, and then decline steadily. The effects of round off error can also be observed in these calculations. Table 11 summarizes the main results associated with these three causes of death for males.

The Value of a Life Table Without Neoplasms for Females

The value of a life table calculated for a male should differ from that calculated for a female because, typically, females live longer and earn less. Both of these characteristics would tend to decrease annual consumption and possibly the value of life, $V_{\dagger\dagger}$. To make comparisons with male life tables more relevant, however, it will be assumed here that females earn exactly what males do, i.e., $24,000 at age 50. (Such an assumption would be appropriate also for married couples, although the total amount presumably would differ from what is assumed here.)

Table 12 gives the life table for American females as calculated by Preston (1972) and (1976). Table 13 presents the estimated life table assuming neoplasms are eliminated, and table 14 summarizes the principal features of these two life tables. Even though annual consumption, $\bar{C}$, is less for females than for males, due to the greater expected life span, the maximum value of a female life, $V_{\dagger\dagger}$, at age 25, is 2.3% larger than that calculated for males. Table 15 gives the compensating surplus and equivalent surplus associated with a neoplasm free life. Compared to males, females would value such a life table somewhat less (except at age 20), but broadly, such differences do not appear significant. Although compensating surplus and equivalent surplus for a life without cardiovascular disease or motor vehicle fatalities is not calculated here for

TABLE 10. COMPENSATING AND EQUIVALENT SURPLUS FOR A LIFE TABLE WITHOUT MOTOR VEHICLE FATALITIES: MALES

Age	COMPENSATING SURPLUS		EQUIVALENT SURPLUS	
	Yearly Annuity Payment	Expected Net Present Value	Yearly Annuity Payment	Expected Net Present Value
20	566	11,740	585	12,077
25	429	8,686	440	0,872
30	405	7,923	415	8,082
35	294	5,526	299	5,605
40	304	5,405	309	5,485
45	313	5,234	319	5,313
50	234	3,620	237	3,661
55	269	3,808	273	3,857
60	289	3,710	294	3,762
65	152	1,747	153	1,759
70	158	1,605	159	1,616
75	164	1,463	165	1,473
80	142	1,081	143	1,090
85	188	1,189	189	1,200
90	89	507	89	510
95	-21	-113	-21	-113
100	-19	-95	-19	-95

Assumptions: $\beta = 0.20$
$R = (1.05)^{-1}$
Maximum earnings = $24,000 (1981)

TABLE 11. SUMMARY, THREE CAUSES OF DEATH FOR MALES

	Original Table	Without Fatal Cardiovascular Diseases	Without Fatal Neoplasms	Without Motor Vehicle Fatalities
Expected life span at age				
20	72.1	82.4	74.3	72.7
40	73.9	84.2	75.9	74.1
60	78.4	87.6	79.9	78.5
80	88.6	94.7	89.0	88.6
Annual Consumption	$15,744	$15,448	$15,683	$15,761
Fraction of all Deaths		56.88%	15.70%	2.67%
Value of Life at Age 25 (millions)	$1,6420	$1.6796	$1.6567	$1.6522
Maximum Compensating Surplus		$166,509 (at age 65)	$48,838 (at age 60)	$11,740 (at age 20)
Maximum Equivalent Surplus		$702,244 (at age 85)	$60,435 (at age 60)	$12,077 (at age 20)

TABLE 12. A LIFE TABLE FOR FEMALES

i	Age	$q_i^*(o)^{(1)}$	$DLY_i(o)$	$DE_i(o)$	Value of Life $V_{\dagger\dagger}$ (millions)
1	20	1.000	4.2597	2.7702	1.6620
2	25	0.996	4.1771	3.1782	1.6853
3	30	0.992	4.0712	3.3063	1.6676
4	35	0.986	3.9435	3.2196	1.6174
5	40	0.977	3.7914	3.0017	1.5437
6	45	0.963	3.6144	2.6465	1.4458
7	50	0.943	3.4074	2.1611	1.3230
8	55	0.914	3.1701	1.5546	1.1761
9	60	0.873	2.8997	0.8291	1.0047
10	65	0.813	2.6035	0	0.8127
11	70	0.727	2.2886	0	0.7144
12	75	0.611	1.9568	0	0.6109
13	80	0.459	1.6255	0	0.5033
14	85	0.280	1.2862	0	0.4015
15	90	0.089	1.1493	0	0.3589
16	95	0.0157	1.0799	0	0.3371
17	100	0.0016	1.0000	0	0.3122

(1) As calculated by S.H. Preston (1972) and (1976).

Assumptions: $\beta = 0.20$
$R = (1.05)^{-1}$
Maximum earnings = \$24,000 (1981)

TABLE 13. A LIFE TABLE FOR FEMALES THAT ASSUMES ALL NEOPLASMS ARE ELIMINATED

i	Age	$q_i^*(n)^{(1)}$	$DLY_i(n)$	$DE_i(n)$	Value of Life V_{tt} (millions)
1	20	1.000	4.3060	2.7905	1.6743
2	25	0.997	4.2323	3.2041	1.7010
3	30	0.993	4.1417	3.3385	1.6890
4	35	0.988	4.0300	3.2597	1.6447
5	40	0.982	3.8907	3.0506	1.5763
6	45	0.972	3.7235	2.7029	1.4826
7	50	0.960	3.5230	2.2205	1.3624
8	55	0.940	3.2886	1.6086	1.2160
9	60	0.911	3.0139	0.8652	1.0414
10	65	0.866	2.7039	0	0.8410
11	70	0.794	2.3718	0	0.7377
12	75	0.6889	2.0180	0	0.6278
13	80	0.539	1.6606	0	0.5165
14	85	0.347	1.3095	0	0.4074
15	90	0.1180	1.1617	0	0.3614
16	95	0.0224	1.0875	0	0.3383
17	100	0.0025	1.000	0	0.3111

(1) As calculated by S.H. Preston (1972) and (1976).

Assumptions: $\beta = 0.20$
$R = (1.05)^{-1}$
Maximum earnings = $24,000 (1981)

TABLE 14. SUMMARY CHARACTERISTICS FOR FEMALES

		Original Life Table	Neoplasms Eliminated
Expected Life Span at Age	20	70.2	81.1
	40	79.2	82.0
	60	82.2	84.1
	80	89.2	89.5
Consumption $\bar{C}$		\$15,608	\$15,552
Fraction of All Deaths			15.24%
Value of Life at Age 25 (millions)		1.6853	1.7010
Maximum Compensating Surplus			\$42,124 (at age 55)
Maximum Equivalent Surplus			\$48,785 (at age 55)

TABLE 15. COMPENSATING AND EQUIVALENT SURPLUS FOR A LIFE TABLE WITHOUT NEOPLASMS: FEMALES

Age	COMPENSATING SURPLUS		EQUIVALENT SURPLUS	
	Yearly Annuity Payment	Expected Net Present Value	Yearly Annuity Payment	Expected Net Present Value
20	767	16,504	809	17,233
25	934	19,752	997	20,813
30	1,229	25,455	1,343	27,265
35	1,550	31,231	1,728	34,063
40	1,838	35,754	2,091	39,650
45	2,102	39,129	2,432	44,072
50	2,344	41,278	2,768	47,171
55	2,562	42,124	3,078	48,785
60	2,687	40,485	3,259	47,249
65	2,636	35,633	3,185	41,455
70	2,497	29,616	2,985	34,163
75	2,173	21,922	2,535	24,797
80	1,527	12,675	1,699	13,805
85	1,286	8,416	1,406	9,042
90	761	4,418	802	4,612
95	430	2,626	500	2,702
100	-54	-273	-54	-273

Assumptions: $\beta = 0.20$
$R = (1.05)^{-1}$
Maximum earnings = $24,000 (1981)

females, the age pattern of the incidence of these causes of death, table 16, indicates that they should be similar to those calculated for males, table 6.

Discussion

Arthur (1981) in his analysis of value of life, using life cycle modeling of consumption, defines a quantity he calls the marginal consumption equivalent, CE, to measure the value of one life over another. It is essentially the difference between the calculated "value of life" for one table and that for the other table.[4] The difference between the two values of life, is from equation (B10),

$$CE_t = V_{tt}(n) - V_{tt}(o)$$

$$= (\bar{C}_n \; DLY_t(n) - \bar{C}_o \; DLY_t(o)) \; \frac{1-\beta}{\beta}$$

$$+ (DE_t(n) - (DE_t(o)) \quad .$$

For small changes in the life table, $\bar{C}_n$ will be approximately equal to $\bar{C}_o$ and so the marginal consumption equivalent would be approximated by the expression

$$CE_t \cong \frac{\bar{C}_o \; (1-\beta)}{\beta} \; (DLY_t(n) - DLY_t(o))$$

$$+ (DE_t(n) - DE_t(o)) \qquad \text{(B16)}$$

and for $t = 1$,

$$CE_1 \cong \frac{\bar{C}_o \; (DLY_1(n) - DLY_1(o))}{\beta} \qquad \text{(B17)}$$

TABLE 16. AGE PATTERNS OF INCIDENCE FOR THREE CAUSES OF DEATH AMONG FEMALES (CONDITIONAL ON LIVING AT AGE 20)

Age	Cardiovascular Disease	Neoplasm	Motor Vehicles
20	0.07%	0.28%	9.85%
25	0.10%	0.44%	6.46%
30	0.17%	0.89%	6.24%
35	0.32%	1.61%	6.24%
40	0.56%	2.80%	6.46%
45	0.95%	4.54%	6.78%
50	1.69%	6.69%	6.89%
55	2.72%	8.49%	8.32%
60	4.71%	10.37%	8.21%
65	7.68%	12.73%	9.19%
70	11.39%	13.49%	8.32%
75	16.15%	13.57%	7.55%
80	20.22%	12.20%	5.69%
85+	33.27%	11.91%	3.83%
Fraction of all Deaths (age 20)	64.5%	15.68%	0.94%

Reference S.H. Preston (1972) and (1976).

Comparing equation (B17) with equation (B15) when $t=1$, CE_1 will be equal to TCS_t. After retirement when $DE_t(n) = DE_t(o) = 0$,

$$CE_t \cong \frac{\bar{C}_o \ (1-\beta)}{\beta} \quad (DLY_t(n) - DLY_t(o))$$

and so CE_t would be less than compensating surplus by the factor $(1-\beta)$. For values of t between 1 and retirement, CE_t could be either larger or smaller than, TCS_t. Table 17 compares these two measures for the large changes in the life table corresponding to the elimination of cardiovascular disease, neoplasms and motor vehicles for males. The two measures are most similar for motor vehicles where the new life table is most nearly like the original one.

Perhaps the most important property identified in this analysis is that a new life table will be most valued by that age group that is at the highest risk. While this has clear intuitive appeal, this property is not obvious from a simple examination of how "value of life," V_{tt}, changes with age. In the examples used here, value of life was at a maximum at age 25, and for this reason, one might assume that this age group would also have the highest willingness to pay for a new and better life table. This is not necessarily true. In fact, for cardiovascular diseases and neoplasms, willingness to pay for a new life table without these sources of death was a maximum in the age range of 55 to 85 when measured by either compensating surplus or equivalent surplus.

Effects of Latency

The assumption that all neoplasms or all cardiovascular diseases can be eliminated is unrealistic, particularly in the context of environmental improvements where survival rates are often increased only slightly (on the order of 1 in 100,000). Table 5 earlier addressed this problem by calculating the compensating surplus for a life table that assumes 1 person in 100,000 could be immunized against cardiovascular disease. The

TABLE 17. COMPARING COMPENSATING SURPLUS WITH ARTHUR'S "MARGINAL CONSUMPTION EQUIVALENT FOR MALES", 1981

Age	Cardiovascular Disease		Neoplasm		Motor Vehicles	
	TCS_t	CE_t (1)	TCS_t	CE_t	TCS_t	CE_t
20	49,908	23,270	14,833	10,330	11,740	13,461
40	109,552	97,148	34,223	49,983	5,405	6,261
60	163,458	180,835	67,621	40,904	3,710	3,757
80	146,859	211,096	19,267	14,922	1,081	1,565

(1) CE is Arthur's marginal consumption equivalent.

values for willingness to pay were, of course, very much smaller--a few dollars at best--but if the population at risk is large, then the total value of such a life table to society would be very large indeed. A similar calculation for neoplasms is given below, where it is again assumed that 1 male in a hundred thousand can be immunized against cancer.

Age	20	30	40	50	60	70	80
TCS_t (dollars)	0.16	0.25	0.38	0.46	0.55	0.43	0.21

These values are about four to five times smaller than that calculated for cardiovascular disease, and reflects the fact that a cancer death is 3.6 times less likely. Total willingness to pay among 100,000 persons aged 60, so that exactly one will have a cancer free life would be 100,000 x 0.55 = $55,000.00. Note that this is not the same as willingness to pay to reduce the number of cancer deaths by one, since that one person with the cancer free life might have died (and indeed is likely to have died) of other causes. Since cancer represents 15.9% of all male deaths at age 60, (see table 18) willingness to pay to reduce the number of cancer deaths by 1 would be approximately valued at

$$\$55,000/0.159 = \$349,900$$

For other age groups, this value would be less.

The value of a male life, V_{tt}, at age 60 is $918,900.00 from table 1. This represents the willingness to pay to avoid a statistical death that occurs immediately. Presumably the difference between this $918,900.00 and willingness to pay to reduce the number of cancer deaths by one, $349,900, is due to a latency factor, since the 60 year old male is not likely to die of cancer immediately.

TABLE 18. PERCENT OF MALES DYING FROM SPECIFIC CAUSES BY AGE AND CONDITIONAL ON BEING ALIVE

Age	Percent of Population[1]	Cardiovascular Disease	Neoplasms	Motor Vehicles
20	6.71	59.37	16.22	2.33
25	5.79	59.86	16.31	1.95
30	5.79	60.032	16.38	1.71
35	6.31	60.08	16.44	1.51
40	6.40	61.229	16.50	1.35
45	5.88	61.79	6.53	1.21
50	5.42	62.236	6.52	1.08
55	4.65	63.09	6.30	0.92
60	3.89	63.98	5.90	0.84
65	3.04	65.19	15.09	0.76
70	2.44	66.76	13.85	0.66
75	1.62	68.52	12.28	0.57
80	0.93	70.48	10.40	0.45
85	0.42	72.20	8 . 6	0.29

SOURCE: Preston, S.H. et al. (1972). These numbers represent US population in 1964.

[1]The fraction of males under age 20 (circa 1964) is 40.71%. Thus, the number of males in the 20-24 age bracket conditional on living to age twenty is 6.71/(1-.4071) = 11.32%.

Additional Concerns

The examples used in this report all used a common set of assumptions:

- the utility function of the form C_i^β where β = 0.2,
- a subjective discount factor and a discount factor for money that is equal to $(1.05)^{-1}$,
- the availability of a perfect market for annuities and insurance, and
- wages such that a person earns a maximum amount at age 50 of $24,000.

The values chosen for β and for the maximum earnings above will alter the value of life, V_{tt}, compensating surplus and equivalent surplus in a simple and direct fashion. The other common assumptions impact these calculations in a more subtle and indirect way and generalizations are not obvious. These results therefore should be viewed as illustrative of a methodology, and not as actual estimates. This is particularly true for values calculated for age groups over 80. First, quality of life and other health issues are more important for older people, and this factor is not reflected in these calculations. Further, the assumption that all people die by age 105 distorts a life table somewhat, particularly under the scenario that cardiovascular diseases have been eliminated. Finally the life tables used in these examples are likely to be more uncertain at these ages -- after all, a life table free of cardiovascular disease is a hypothetical construct. This uncertainty is compounded by the fact that round off error is more important at these age groups when the survival probabilities, q_i^*, are small.

In summary, this appendix has developed a methodology for valuing one life table over another and has illustrated this technique using several scenarios. Further, the effects of latency have been identified by comparing the willingness to pay to avoid an immediate death to the willingness to pay to avoid a cancer death (etc.) that will occur sometime in the future.

NOTES

1. If the real and subjective discount rates differ, then consumption will increase or decrease over time depending on whether D/R is greater or less than 1.0.

2. Actually the preferred measure for valuing welfare change (see Freeman 1979) is equivalent and compensating variation. However, Freeman shows that when there are only two alternatives available, compensating surplus is identical to compensating variation, and equivalent surplus is identical to equivalent variation.

3. By equation B4,

$$\bar{C}_o = \left(\sum_{i=1}^{T} q_i^*(o)\, R^{i-1}\, y_i) \,/\, DLY_o \right) ,$$

$$= \frac{((1.0)(1.05)^0(.28)+(.991)(1.05)^{-1}(.59)+\ldots)\ 24,000.00}{((1.0)(1.05)^0 + (.991)(1.05)^{-1}+\ldots)}$$

$$= 15,744.00$$

4. Arthur's analysis differs from the analysis of Cropper (1982) and Shepard and Zeckhauser (1982) by putting a consumption constraint on society as a whole rather than on the individual. Thus, while similar, Arthur's results are not directly comparable to the results obtained by these other authors or to the results presented here.

REFERENCES

1. Arthur, W.B. (1981). "The Economics of Risk to Life," American Economic Review, pp. 54-64.

2. Cropper, M.L. (1982). "Valuing Risks to Life," preliminary report.

3. Freeman, A.M. (1979). The Benefits of Environmental Improvement. Resources for the Future, Inc., Johns Hopkins University Press.

4. Preston, S.H. (1976). Mortality Patterns in National Populations, Academic Press, New York.

5. Preston, S.H., N. Keyfitz and R. Schoen (1972). Causes of Death: Life Tables for National Populations. Seminar Press, New York,

6. Shepard, D.S. and R.J. Zeckhauser (1982). "Life Cycle Consumption and Willingness to Pay for Increased Survival." The Value of Life and Safety, M.W. Jones-Lee ed. North-Holland Publishing Company.