APPENDIX D

Sensitivity Studies

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ABBREVIATIONS AND ACRONYMNS

AWQC	Ambient Water Quality Criteria
BIF	Bunker Hill Box inclusion factor
NRC	National Research Council
RI	remedial investigation
RLP	relative loading potential
TEM	total effective metal mass
TMDL	total maximum daily load

D.1.0 INTRODUCTION

Models are created to mimic or approximate systems of varying types and complexity, be they physical, environmental, social, economic, or otherwise. In many cases, physical experimentation on the actual system is too complex, too time consuming, too expensive, or even impossible. In these cases we revert to mathematical or conceptual models to diagnose or forecast system behavior.

A mathematical model is a set of equations, parameters, and input variables that intends to characterize the system being studied. Uncertainty is created by errors of measurement, absence of information, limited or poor understanding of processes. A model may also have to deal with intrinsic (i.e., aleatory or random) variability in the system. This uncertainty limits the confidence one places in the output or response of the model.

Good practice necessitates that we attempt to evaluate the confidence in model output or response by assessing the uncertainty associated with the modeling process and outcomes. One approach to doing so is through sensitivity analysis. Sensitivity analysis is the study of how the variation in model output or response depends on, or may be apportioned among, variation in the equations, parameters, and input variables that constitute the model. Put simply, "sensitivity analysis studies the relationship between information flowing in and out of the model" (Saltelli 2006).

The purpose of sensitivity analysis is to determine which factors have the greatest effect on measures of performance. Sensitivity analyses are used to assess how robust the results of a model are to assumptions about the data and methods that were used, or to parameter estimates. While uncertainty may arise in estimating the values of input parameters, if the output forecasts change only gradually or moderately with changes of the input variable, then the effect of uncertainty on the input on uncertainty in the output is itself moderate.

The influence of uncertainty on the input parameters on uncertainty on the output can be approximated by the first-order relationship,

$$Var(y) \approx \sum_i \sum_{j \neq i} \frac{\partial y}{\partial x_i} \frac{\partial y}{\partial x_j} Cov(x_i, x_j) \tag{1}$$

in which y is the output forecast and the x_i, x_j are the input parameters. The term $Cov(x_i, x_j)$ s the covariance of the two input parameters, which becomes the variance when $i = j$, and is zero if uncertainties in the two parameters are independent. Thus, for the case of independence among all the uncertainties in all the input parameters, this relationship reduces to

$$Var(y) \approx \sum_i (\frac{\partial y}{\partial x_i})^2 Var(x_i) \tag{2}$$

As a consequence, if the derivative of the output with respect to the input—that is, the change in output for an incremental change in the input—is moderate given the variance of the uncertainty in the input, then one can be relatively confident that the uncertainties in the input lead only to proportionate uncertainties in the output.

D.2.0 MODELING ASSUMPTIONS

A summary of assumptions underlying the model is discussed here. Assumptions are divided into three groups having to do with the conceptual model of the overall system, loadings, and remedial effectiveness. This section also summarizes the reasoning behind the assumptions, based on the more complete discussion in the main text and Appendices B and C of the technical memorandum.

This section first addresses the objectives of the modeling effort in order to set the context for the conceptual model. It then addresses the assumptions made in constructing the conceptual model, dividing these among the systems model, contaminant loading, and remedial effectiveness. Then it addresses the uncertainty analysis.

D.2.1 GOALS AND PERFORMANCE MEASURES

Modelers and philosophers of science have long debated whether models can be proven "to be true" in an epistemological sense (e.g., Oreskes et al. 1994). Today, most of these workers would probably agree that a model cannot be proven true, but is rather corroborated. Corroboration means that the model has passed a series of tests of logic, internal consistency, and calibration to the world (i.e., a capacity to explain or predict natural processes in a convincing yet parsimonious way) (Saltelli 2006). A validation of the principal assumptions of the present model is argued on the first two grounds, logic and consistency. The database on changing conditions in the Coeur d'Alene basin as a result remedial actions is yet too sparse to allow direct calibration of a model to predictions of future behavior.

Validation of conceptual model assumptions must consider whether those assumptions are corroborated at an appropriate level of detail to fit the purpose intended. The model is necessarily a simplification or abstraction of reality. The choice of which components to include in the model, or whether one has chosen, among multiple abstractions that describe a system equally well, the model that is most suitable depends on the purpose at hand.

The purpose of a model may be diagnosis or prognosis, that is, understanding the real system or predicting its behavior. In the present case, the purpose is prognosis. While the model is deterministic, an uncertainty analysis is used in an attempt to identify the sources and magnitudes of uncertainties entering the model and to track how those uncertainties propagate to the model's outputs. To do this, the model uses mathematical techniques from probability theory, but the conceptual model itself is purely deterministic. That is, the model generally uses probability theory to track uncertainties in the parameter estimates entering the model. Because the model is a "linear system" and thus simple in its behavior, it is also simple in the way

uncertainties are propagated through the model. The assumptions of the uncertainty analysis are discussed below.

D.2.2 CONCEPTUAL MODEL ASSUMPTIONS

The conceptual modeling approach used in the predictive analysis for the Coeur d'Alene basin is based on a straight-forward, deterministic accounting concept of how dissolved metal loadings leaching from upstream source areas combine to generate downstream loadings at specific locations along the river. The modeling makes assumptions on the rates of loading from specific source types and the proportional effectiveness of remedial alternatives in reducing those loading rates. The loadings from all the source types are then summed to predict downstream loading. It is a simple linear model, much like a bookkeeper's spreadsheet.

D.2.2.1 Systems Assumptions

Dissolved zinc is used as the metal of concern and as an indicator for other metals.[1] The predictive analysis is limited to dissolved metals because Ambient Water Quality Criteria (AWQC) apply only to dissolved metals. The current model uses dissolved zinc as an indicator for total dissolved metals. Other metals, such as cadmium or lead, can be incorporated in the predictive analysis as necessary. Dissolved zinc is used as an indicator for the following reasons: zinc is the most ubiquitous of the metals in the study, zinc occurs at the highest measured concentrations and has the highest ratios of average measured concentration to AWQC, zinc is mobile compared to other metals, and other dissolved metals typically correlate with dissolved zinc.

Use of metal loading rather than concentration as the measure of downstream transport. For surface water, the performance goal driving development of remedial alternatives has been to reduce the concentrations of dissolved metals to meet AWQC. The model limits attention to dissolved metals, because current AWQC apply only to dissolved metals. AWQC are functions of water hardness, which generally correlates with stream discharge; thus AWQC depend on discharge and location. Compliance with AWQC was quantified using *loading capacities*. Loading capacities are the product AWQC concentrations and stream discharge. In practical terms, AWQC are met if loadings are below total maximum load daily load (TMDL) loading capacities, thus simplifying the model without loss of accuracy.

[1]An exception to the use of dissolved zinc as an indicator is total lead in the lower basin, and remedial alternatives in the lower basin have been specifically developed to deal with total lead.

The distance of an upstream source from its downstream effect is ignored when accumulating downstream volumes. The model is a simple accounting scheme, which adds up all sources upstream in calculating downstream loadings. The presumption is made that metal loadings that enter upstream eventually make their way downstream.

Stream discharges were correlated with AWQC using a continuous interpretation of the TMDL "flow tiers" (USEPA 2000c). Because AWQC are functions of hardness, it is assumed that the relationships between stream discharge and water hardness will continue into the future and not be significantly affected by remedial action. For consistency, the same equations that relate AWQC to hardness as used for TMDL loading capacities (USEPA 2000c) were used in the analysis. In practical terms, therefore, AWQC are met if loadings are below TMDL loading capacities.

Deeper aquifer material is represented as a zinc source. The analysis has included the volume and effect of zinc in deeper aquifer material within the collective unremediated source type, "deeper impacted floodplain sediments." If the true zinc loading from deeper aquifer material greatly exceeds the estimate, then for each action alternative, the expected values of the remediation factor, loadings, and load ratios would be systematically underestimated. This underestimation would likely occur in the "short term," on the order of perhaps a century or less. Further consideration of this potential effect should be included in future updates of the model.

Groundwater metal load not discharging to surface water only affects load contributions between monitoring locations. The practical implication is that impacted groundwater does not affect surface water quality until it discharges to surface water. The load carried by such groundwater could cause errors in the estimates of source contributions within affected stream reaches between monitoring locations. In general, dealing with these kinds of groundwater effects would require a more refined consideration of groundwater-surface water interactions than included in the current model.

AWQC will not change. AWQC are functions of hardness. It is assumed that the relationships between stream discharge and water hardness will continue into the future and not be significantly affected by remedial actions. The relationship between stream discharge and water hardness is accounted for in the model. Potentials for future regulatory changes could be considered in interpreting the results or in future changes to the model.

D.2.2.2 Loading Assumptions

Loading is proportional to total metal mass. Absent direct measurements of the loading rates of individual source areas within a source type, some assumption must be made about the relationship of source size to the loading of dissolved metal it generates. Since within a source type the physical processes generating the loadings are presumably similar, it is reasonable to

presume that larger sources generate larger loadings. Both physical size and the concentration of metal within the source might be reasoned to correlate with loading. The most parsimonious combination of these two is their product, volume times concentration, which is the total amount of metal within the source. This makes the presumption that the total mass of metal within a source is proportional to the metal mass available for leaching, which again seems reasonable. Much more complex models of loading could be proposed, but would not themselves be supported by observational data. Thus, the simple model was adopted.

Source types represent spatial averages. The model aggregates all upstream sources of a given type into a single element. For the upper basin, these source types are tailings piles, waste rock piles, adits, and tailings-impacted floodplain sediment. For the lower basin, these source types are river banks and levees, riverbed sediments, wetland sediments, lateral lake sediments, other floodplain area sediments, and Cataldo-Mission Flats dredge spoils. The model assigns single values of the parameters—RLP_j, V_j, and R_{ij}—to each source type, j, despite the fact that there is undoubtedly variation within and among individual sources.

The variation within and among sources is measured as part of the coefficients of variations of these parameters. That is, the uncertainties ascribed to the parameter values reflects both intrinsic (spatial) variability and knowledge (epistemic) uncertainty. Representing spatial variation in a model as part of a coefficient of variation on lumped parameters is common practice in the earth sciences, since spatial variation exists on any geographic scale, from basin averages (as used in the analysis) to site specific source averages (e.g., Jornel and Huijbregts 1978).

The estimated proportionality of zinc loading to total zinc mass is based on annual averages over a water year so as to average out daily and seasonal fluctuations. Statistical homogeneity for time was obtained by making the time of interest one water year. Seasonal and shorter term effects are thus integrated and averaged-out, as discussed in Appendix B. The temporal variation was incorporated as part of the intrinsic variability of the loading parameters.

D.2.2.3 Remediation Effectiveness Assumptions

Post-remediation loadings are predicted from the pre-remediation loadings using a "remediation factor," R_{ij}, in which i is the index of remedial alternative and j is the index of source type. The remediation factor is an estimate of the ability of a remedial alternative to reduce future metal loadings and is the key to the post-remediation estimates.

Load reduction for a given source type is proportional to volume of source remediated. This assumption follows from the assumption that loadings are proportional to total effective metal mass (*TEM*) within a given source type. Presuming specific source types to have a more or less constant concentration of metal within themselves, the reduction of loading caused by a

remedial action should be proportional to the change in volume of the source caused by that action. Arguments might be made that the reduction of loading could plausibly be related to area or linear extent of the source, rather than volume. Even were that true, given that within a class the source geometries are relatively the same (e.g., sedimentary dispositions are all relatively tabular), all these geometric ratios yield relatively similar proportions. Errors potentially introduced by this assumption are not thought to be of practical importance because of the spatial averaging effect, the way source types were defined and aggregated, and how remedial effectiveness was characterized as an aggregate effect for each source type and remedial technology.

Because of the effect of source depletion, remediation factors are functions of time. Estimates of natural recovery were based on a model of source depletion that describes the reduction in metal loading and concentration over time as an exponential function of a decay rate (Appendix B). The decay rate is assumed to be constant over time. The decay rate is an approximation that averages effects across source types as a function of *TEM*. The model uncertainty inherent in this averaging is explicitly included in the uncertainty analysis.

The remediation factor for a given remedial action represents spatial averages for a given source type. Intrinsic variability in time or space is addressed in the uncertainty analysis.

D.2.3 UNCERTAINTY ANALYSIS ASSUMPTIONS

The predictive analysis is based on a deterministic conceptual model of contaminated sources, metal loadings, and remedial effectiveness. The impetus for the model originated in the natural complexity of the basin and from the need to combine a large number of considerations, calculations, and parameter estimates in making forecasts. Many uncertainties enter this calculation, and not all of these consolidations and parameters are known with precision. So, while the model generates best estimate forecasts, these forecasts necessarily contain uncertainty.

An uncertainty analysis was incorporated within the model to ensure that uncertainties entering the model be quantified and then translated into uncertainty on resulting forecasts. The predictive analysis uses a simple probabilistic approach to capturing uncertainties and propagating their combined effect through to forecasts. Best estimates, standard deviations, and correlations are assessed for the input entering the model; and best estimates, standard deviations, and correlations are calculated for the output.

The standard deviations associated with the output allow quantitative conclusions to be drawn about the degree of uncertainty in the forecasts. The model itself calculates uncertainties as represented in the first two probabilistic moments of the predicted values. The first two moments are the mean or expected value and the variance (or its square root, the standard

deviation). For operational convenience, sometimes the ratio of the standard deviation to the mean, referred to as the coefficient of variation, is used as a metric.

Note that the information contained in the first two moments is not enough to allow precise statements to be made about probabilities of exceedance or on interval estimates, for example, the probability that a loading at some time in the future will exceed a certain level. For these purposes, some assumption must be made on the shape of the probability distribution, which the first moments, the mean and the variance, define. In most cases in the present model, these distributions are assumed to be *lognormal* , that is, to have a probability distribution that is *normal* or traditionally bell-shaped on the logarithm of the variable. There is good reason for making this assumption, as discussed below, but the sensitivity of output to this assumption is also tested.

The predictive analysis combines existing information on the basin with scientific understanding of environmental processes, but neither the existing information nor our scientific understanding of environmental processes is perfect. As a result, the scientific and professional judgment of experts is called upon to interpret data and to help estimate parameter values. This process, too, introduces uncertainties, which the predictive analysis attempts to capture.

D.2.3.1 Intrinsic (Aleatory) Variability vs. Epistemic Uncertainty

The uncertainties entering the model are categorized in two types: intrinsic variability (sometimes called, *aleatory* uncertainty), and lack of knowledge or understanding (sometimes called *epistemic* uncertainty). The distinction is important, because aleatory uncertainties may average out over time and space, but epistemic uncertainties do not. That is, in some circumstances they behave differently.

Aleatory uncertainty has to do with the variation of natural processes in time and space. For example, in some years there is a great deal of rainfall, while in others there is not. Remedial effectiveness depends on stream flow, and as a result it, too, will differ from one year to another. At some locations within a source area metal concentrations are high, while in other areas they are low. Load contributions are assumed to depend on total effective metal mass, and as a result, they will differ from point to point within the source.

Epistemic uncertainty has to do with limited information. For example, there may be too few data from which to precisely estimate the total metal loading from a given source type, so judgment must be used. Post-remediation loadings also depend on hypothesized, potential future remedial actions, which are limited to conceptual designs at best, and corresponding conditional remedial effectiveness, for which there are no measurement data. Estimates of these hypothetical potential future actions and conditions must therefore be based on professional interpretation and judgment. In theory, errors in the judgmental estimates would cause errors in

the (theoretical) downstream prediction. It should be noted, however, that as additional information became available during subsequent remedy selection and implementation, the epistemic uncertainty would decrease, and so would the potential error in relevant estimates.

The predictive analysis accommodates both types of uncertainty, *aleatory* and *epistemic*, respectively. Whereas, in some modeling applications aleatory results are reported as frequencies and epistemic uncertainties are reported as confidence (actually probability) bounds on those frequency curves, in the present application the two types of uncertainty are combined into a single probability distribution, that is, they are convolved.2

D.2.3.2 Distributional Forms (Lognormal Assumption)

The predictive analysis uses second-moment information (means and variances) on input parameters and other uncertainties and propagates those to second-moment predictions on the output forecasts. However, to make statements about non-exceedance probabilities, or the probabilities of achieving AWQC standards, or probabilities of bounding intervals and thresholds of any kind, some assumption usually must be introduced about the shape of the full probability distribution of the prediction.

Absent an assumption on the shape of the probability distribution, the best commonly available bound on the probability of a variable exceeding some excursion from the mean—that is, the probability that the variable would deviate from the mean by at least some interval—is provided by the Chebychev bound, which is $P(|x - E[x]| \geq t) \leq Var(x)/t^2$. In other words, the probability of the random variable *x* being at least as far away from its mean as *t,* is always less than or equal to the variance of *x* divided by the square of *t* irrespective of the probability distribution of *x*. As can be seen in Table D-1, which shows a comparison to the normal and lognormal distributions, the Chebychev bounds are not usually close to the actual exceedance probabilities and, therefore, are often of limited usefulness.

Left (lower) side exceedance probabilities for normal, a selection of lognormals, and triangular distributions are shown in Figure D-1. Up to about two standard deviations, the probabilities are reasonably close, meaning that they are reasonably robust to the assumption of a distributional shape. Beyond two standard deviations, the differences become larger. Thus, for exceedance probabilities as large as about 0.01, the results are not particularly sensitive.

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2 Mathematically, if *y* is a forecast variable with aleatory frequency distribution $f(y \mid \theta)$, and θ has epistemic uncertainty represented by $f(\theta)$; then, the marginal or predictive distribution of *y* is $f(y) = \int_\theta f(y \mid \theta) f(\theta) d\theta$.

In the predictive analysis, the variables RLP_j, R_{ij}, and V_{ij} are each assumed to be lognormal distributions, so that exceedance probabilities can be directly calculated. That is, the logarithms of the variables are assumed to be normally, or bell-shaped, distributed. As a direct result, the calculated downstream loads are approximately lognormally distributed.

This assumption can be justified on at least five grounds:

- First, historical stream concentration and load data are statistically shown by goodness-of-fit tests to be lognormal in data from the remedial investigation (RI) (USEPA 2000a), and other basin water quality data (Figure D-2). This is consistent with the assumption that RLP_j, R_{ij}, and V_{ij} are lognormally distributed.

- Second, many earth science parameters are empirically observed to obey lognormal probability distributions, particularly geometric variables and trace mineral concentrations, including chemical pollutants. As a result the assumption of lognormality is common in the geological literature (e.g., Davis 1986; Jornel and Huijbregts 1978; Agterberg 1974; Ott 1995; and Corbett et al. 2000).

- Third, the theory of Successive Random Dilutions (SRD) predicts that pollutant concentrations in surface water should approach lognormal distributions under rather general conditions (Ott 1995). As Ott summarizes, the SRD theory "*offers a simple yet surprisingly general explanation of why approximately lognormal distributions arise so often in environmental phenomena*" (p. 223). Ott 1995 also provides and references numerous empirical examples of approximate lognormality in water quality, as well as other environmental media (see also Vogel, Rudolph, and Hooper 2005).

- Fourth, the basic calculations of the predictive analysis involve multiplications of uncertain parameters. By the central limit theorem, the products of almost arbitrarily distributed variables tend toward lognormality as the number of terms in the products rise. Furthermore, in a related earth science application, Barouch et al. (1986) have shown that the sum of lognormal variables, as for example summing lognormally distributed loading contributions of several sources, is approximately itself lognormal (see also Leipnik 1991 and Wu et al. 2005).

- Fifth, results neither for non-exceedance probabilities related to remedial effectiveness, (Figure D-3) nor exceedance probabilities related to time (Figure D-4) are particularly sensitive to the distribution assumption within reasonable bounds.

Additional discussion on the lognormality assumption can be found in Appendix A (Section A.2.2) and Appendix B (Section B.3).

D.2.3.3 Spatial and Temporal Averaging Assumptions

In simplifying basin conditions for the purpose of estimating remediation factors over time (which includes the decay rate), the predictive analysis averages temporal variations in stream flow and loadings over water years, and it averages source conditions over sources of each type. In both cases, this averaging takes mean values over time or space, respectively, and ignores the intrinsic variation in the natural phenomena. This might be thought of as smoothing the temporal or spatial signal. The spatial or temporal variations are described by probability distributions having statistical parameters (i.e., expected values and coefficients of variation) over the times and volumes of interest.

D.3.0 INPUT PARAMETERS

Post-remediation loadings—the main output of the modeling effort—are calculated from three considerations and three corresponding parameter estimates. First, pre-remediation loadings are estimated for each of the half-dozen to dozen source types. These are based on the *relative loading potential* for each source type (see Section 2.3 of the technical memorandum).

Second, *remediated volume* and *remedial effectiveness* are directly estimated for each combination of remedial alternative and source type. The post-remediation loading immediately after remediation is calculated as the product of the pre-remediation loading for each source type, the fractional volume of each source material remediated by each alternative, and the effectiveness of a remedial alternative for the respective source type (see Section 2.4 of the technical memorandum).

Third, decline of the post-remediation loading with time due to source leaching is calculated by multiplying the initial post-remediation loading by an exponentially decreasing decay term. However, this rate of decline is directly calculated from the pre-remediation loading and the total effective metal mass and, thus, is a derived variable.

Thus, three (uncertain) parameters are important in calculating post-remediation loading:

RLP_j = *Relative loading potential for source type j*
R_{ij} = *Remedial effectiveness of alternative i for source type j*
V_{ij} = *Volume of source type j remediated by alternative i*

These parameters were the focus of the sensitivity and validation studies.

D.3.1 FORM OF PARAMETER ESTIMATES

Each parameter is estimated by an expected value and a coefficient of variation. The expected value is the best estimate. The coefficient of variation is the uncertainty in the estimate. This uncertainty incorporates both intrinsic variability and epistemic uncertainty.

The principal input parameters of the model together with the basis and rationale of their estimation are discussed in detail in Appendices B and C and summarized in Table D-2. Other parameters enter the calculation, but are assumed to have negligible uncertainty and, thus, are taken as known. It is important to note that the predictive analysis is built on the RI/FS (Remedial Investigation and Feasibility Study) for the basin and thus is predicated on a large amount of empirical information and historical data that do not appear in, but are only referenced from, the documentation for the predictive analysis.

D.3.2 ENGINEERING JUDGMENT

The predictive analysis combines existing information on the basin with scientific understanding of environmental processes, but neither the existing information nor our scientific understanding of environmental processes is perfect. As a result, the scientific and professional judgment of experts is called upon to interpret data and to help estimate parameter values. The requirement to make estimates from limited information based on reasonable assumptions and professional judgment is the rule in environmental engineering and scientific practice. This is particularly true where complex phenomena must be simplified with approximations and future conditions must be predicted. The goal of an uncertainty analysis is to incorporate as explicitly as practical the potential error inherent in these realities.

The use of expert judgment in environmental modeling and risk assessment has gained widespread support in recent years, as evidenced by a series of National Research Council reports on risk analysis in regulatory and policy decision making (NRC 1994, 1996, and 2007), as well as reports by the Nuclear Regulatory Commission (Budnitz et al. 1998), the European Commission (Cooke and Goosens 1999) and others. In the end, there are only three primary ways that parameter estimates are made: on the basis of statistical estimates from empirical data; by reasoning from first principles, that is, by modeling; or from judgment, hopefully of experienced subject matter experts.

Expert judgment is used in many technical assessments and risk analyses, but it is sometimes implicit and undocumented. In a formal elicitation, a range of scientific and technical opinions are presented, uncertainties represented by these differing views and interpretations are incorporated, and the results are documented. Expert elicitations are guided by formal, structured, and documented processes for identifying and quantifying uncertainties. The goal is to incorporate the views of the breadth of the scientific community.

Because of the need for professional judgment, the possibility cannot be precluded of legitimate differences in professional opinion as to exactly how the available information is interpreted to make specific estimates. Uniquely objective estimates that assure unanimous and unqualified acceptance are unrealistic ideals, particularly where viewpoints conflict. The goal of the probabilistic approach is to quantitatively incorporate, as objectively and explicitly as practical, the uncertainty inherent in the estimates. It is also expected that estimates will be refined over time as new information or improved understandings or hypotheses evolve.

D.4.0 SENSITIVITY ANALYSES OF PARAMETERS

Sensitivity analyses were carried on the three principal parameters: relative loading potential, remedial effectiveness, and remediated volume. The assumption of logNormality of uncertainties was also addressed in the sensitivity studies. Other sensitivity studies were also carried out, for example on the rate of leaching over time, but these are not discussed in detail here. URS 2007 provides a detailed compendium of the sensitivity analyses conducted for the predictive analysis.

The expected values and coefficients of variation of these input parameters are detailed in Appendix C, together with the rationale for their estimated values. In brief summary, the coefficients of variation for the principal parameters for the upper basin are presented in Table D-3 (coefficients of variation estimated for the lower basin used the simplified approach documented in Appendix B, Section B.3.3.3 and Appendix C).

There are various ways of measuring sensitivity. The best way depends on the purpose at hand. Consider a simple sum of *n* items,

$$C = \sum_{i=1,n} C_i \tag{3}$$

One way to measure sensitivity is to take the derivative of the total *C* with respect to each of the components, C_i,

$$S_i = \frac{\partial C}{\partial C_i} \tag{4}$$

in which S_i is the sensitivity of C with respect to C_i. This sensitivity is the proportional change in output for an incremental change in input.

Another way of measuring sensitivity is to perturb each component by some fixed fraction of its nominal or expected value,

$$S_i = \frac{\partial C}{\partial C_i} \frac{E[C_i]}{E[C]} \tag{5}$$

in which *E[.]* is the nominal or expected value. In this case, items with high nominal value are deemed to have high sensitivity.

Yet another way of measuring sensitivity is to consider how variable or uncertain the components are and to perturb each component accordingly. Thus, each component might be perturbed by its standard deviation,

$$S_i = \frac{\partial C}{\partial C_i} \frac{SD[C_i]}{SD[C]} \tag{6}$$

in which $SD[C_i]$ is the standard deviation of the variation or uncertainty in C_i. This measures the contribution to the variation or uncertainty in the total contributed by variation or uncertainty in the individual components. Note that this measure of sensitivity is closely related to the first-order second-moment method of propagating uncertainties through a model that is discussed in Section 2.6 of the technical memorandum and referred to in Equation 1 above.

D.4.1 FIRST-ORDER SENSITIVITY DERIVATIONS

Section B.2.2 in Appendix B algebraically derives approximate sensitivity relationships for *post remediation loading* with respect to the principal parameters: *relative load potential, remediated volume, and remediation factor.* These are shown below, in which *i* is an index for the remedial alternative, *j* is an index for the source type, R_0 is the remediation factor immediately post-remediation, and V_j is the pre-remediation volume of source type *j*:

$$\partial R_0 / \partial RLP_j = (\sum_i (R_{ij})(V_{ij}) - R_0 V_j) / \sum_j (RLP_j)(V_j) \tag{7}$$

$$\partial R_0 / \partial V_{ij} = RLP_j (R_{ij} - R_0) / \sum_j (RLP_j)(V_j) \tag{8}$$

$$\partial R_0 / \partial R_{ij} = (RLP_j)(V_{ij}) / \sum_j (RLP_j)(V_j) \tag{9}$$

These sensitivities were also calculated numerically over a wide domain of possible input variables, as reported in the following subsections.

The importance of the sensitivity studies lies in ascertaining how much uncertainty there is in the calculated results as a function of the uncertainty in the input parameters. For example, how much change can one expect in the forecast of zinc concentrations at Pinehurst and Harrison if the input parameters are in error by, say, 10 percent? The derivatives of Equations 7 through 9 show the mathematical relationships between the variability of the remediation factor to the variability to the respective input parameters. As the sections below indicate, the sensitivities are relatively low. For example, a sensitivity of +0.1 means that the output of the predictive analysis

calculation only changes upward by10 percent for a unit change in the parameter value. A sensitivity of -0.1 means that the output changes downward by 10 percent for the same change.

D.4.2 RLP_j, RELATIVE LOAD POTENTIAL

The sensitivities of downstream loadings to relative load potential for the various source types, measured by the derivative, are summarized in Table D-4. An example of one set of calculations is shown graphically in Figure D-5 for the source type, floodplain sediments. Similar graphs were prepared for all source types in the upper basin. These sensitivities range from −0.14 to 0.00, with the exception of the unremoved deeper floodplain sediments, which have sensitivies as high as 0.58.

The way this graph should be interpreted is the following:

- The curves show the expected value of the remediation factor, $E[R_o]$, as a function of the relative load potential, *RLP*, used in the calculations for, in this case, floodplain sediment sources. The remediation factor is the fraction of metal load remaining at Pinehurst immediately after remediation.

- The squares in the middle of the graph are the best estimates of $E[R_o]$ for floodplain sediment sources for the respective remedial alternatives.

- The dashed box shows the 0.05 and 0.95 probability bounds on the estimate of *RLP, which defines the 90% confidence interval, 90%CI, and which has an estimated 90% probability of including the uncertain true value of RLP between these bounds*.

- The sensitivities in Table D-4 are slopes of the curves found by taking the value of $E[R_o]$ at the 0.95 bound on *RLP*, subtracting the value of $E[R_o]$ at the 0.05 bound on *RLP*, and dividing by the difference in the value of *RLP* between those two bounds (mathematically, the values in Table D-4 are the slopes of the secant to each of the curves).

For example, for Alternative 4, this slope is approximately −0.14. The negative sign indicates that $E[R_o]$ decreases as *RLP* increases. This means that a unit uncertainty in *RLP* for floodplain sediments generates an 0.14 unit uncertainty in the predicted loading at Pinehurst and that increasing values of *RLP* causes decreasing values of predicted loading.

The range of uncertainty in the input values of relative loading potential are similarly high, although the relatively high sensitivity for deeper floodplain sediments is somewhat balanced by a low coefficient of variation. This means that, although the sensitivity is high, the uncertainty in the input parameter that is generating that sensitivity is relatively low. Thus, the net result is that the modeling error in the output is modest. Post-remediation loadings are more sensitive to RLPs than to remedial effectiveness or remediation volumes.

D.4.3 *Rij,* REMEDIATION FACTOR

The sensitivities of downstream loadings to remedial effectiveness, as measured in the predictive analysis by its complement, the remediation factor, for the various remedial alternatives and source types, measured by the derivative, are summarized in Table D-5.

The example for floodplain sediments is shown in Figure D-6. These range from 0.02 to 0.30. The range of uncertainty in the input values of the remediation factor measured as coefficients of variation are 0.04 to 0.7. Thus, the uncertainty in remediation factors have a relatively small contribution to the uncertainty in downstream, post-remediation loadings. The one exception to this range of uncertainty is the remedial alternative, excavation and disposal in a repository, which has a coefficient of variation of 1.76 and is part of Alternatives 3 and 4. However, the expected value of the remediation factor for this remedial approach is about 0.01, meaning that the approach is highly effective, and the high coefficient of variation is principally due to a small standard deviation being divided by an equally small expected value. The impact on predictive uncertainty is therefore negligible.

D.4.4 *Vij,* REMEDIATED VOLUME

The sensitivities of downstream loadings to the volume remediated for the various source types, measured by the derivative, are summarized in Table D-6. The example for floodplain sediments is shown in Figure D-7. These range from −0.02 to −0.30.

The range of uncertainty in the input values of the volume remediated is 0.2 to 0.3. Thus the uncertainty in remediated volumes has a relatively small contribution to the uncertainty in downstream, post-remediation loadings.

D.4.5 CORRELATIONS AMONG PARAMETERS

The principal parameters themselves are dependent on multiple uncertain variables, and thus it may be necessary to consider correlation between the variables. Correlation is quantified by the covariance or correlation coefficient, as discussed in more detail in Appendix A.

The correlation coefficient measures the degree of linear correlation between the two variables, and may range between plus and minus one. Independent variables are uncorrelated and (always) have a correlation coefficient of zero. The converse, that variables with a zero correlation coefficient are independent, is not generally true. Given a pair of variables, a correlation coefficient of zero implies no linear correlation. A correlation coefficient of one implies perfect direct linear correlation (i.e., the variables are directly proportional). A correlation coefficient of minus one implies perfect inverse linear correlation (i.e., the variables are inversely proportional). Correlation may be "deterministic" (i.e., functional or causal), probabilistic, or a mix of both. Correlation occurs naturally between variables having the same or related causes and can occur between variables that are functions of common variables.

Correlations Among RLP_j, V_{ij}, and R_{ij}

The combined effect of correlations between RLP_j, V_{ij}, and R_{ij} and how this effect was dealt with in the analysis is detailed in Appendix B, Section B.3.3.1. While there is no functional reason to suspect correlation between relative loading potentials and the volume of source remediated (which is a decision variable), there may be induced correlation. For example, Alternatives 2, 3 and 4 were aimed at the high *RLP* source types, which may induce correlation between V_{ij} and RLP_j for these alternatives. Similarly, while there is little reason to think that the volume of source remediated and the remediation effectiveness are functionally correlated, there may be induced correlation for some alternatives; e.g., Alternative 4 extensively used remedial actions with low R_{ij} for large V_{ij} associated with affected floodplains.

On the other hand, there is some reason to suspect that relative loading potential and remedial effectiveness may be correlated, although there are at present no observational data to suggest the magnitude of a possible correlation. There are also correlations between the RLP_j, which cannot be accurately quantified with available data.

The combined effect of correlations among the *RLP*, V_{ij}, and R_{ij} manifests in the remediation effectiveness, R_o. Remedial effectiveness, R_o, for a given alternative is calculated as the ratio of post-remediation loading to pre-remediation loading for that alternative, which is proportional to the ratio of the relative remediated volume of sources after remediation to the relative remediated volume of sources before remediation. These relative volumes in turn depends on common values of *RLP's*, R_{ij}*'s*, and V_{ij}*'s*. Thus, the numerator and denominator of the ratio used to forecast remedial effectiveness immediately post remediation are positively correlated, as discussed in Appendix B Section B.3.3.1. This correlation tends to reduce the uncertainty in R_o but at the same time also change its mean, because the variables are assumed to be lognormally distributed.

Table D-7 provides sensitivity analysis results of the effect of correlation between pre- and post-remediation relative remediated volumes on the expected value and coefficient of variation of *Ro*. The shaded values of correlation coefficient are those taken as best estimates for analysis, using the analysis methodology detailed in Appendix B Section B.3.3.1. The effect of variation in this correlation coefficient is small in the expected value, and only modest in the coefficient of variation.

Correlation Between Pre-remediation Load L and Remediation Factor R

Higher metal loads at Pinehurst and Harrison are correlated with higher stream discharges. While the concentration of metals at higher discharge decreases, the larger volume of water more than makes up for this decrease in concentration, with the result that the load increases. However, the remediation factors at higher discharges are expected to decrease. Thus, there is expected to be a negative correlation between load and remediation factor *R*.

In the analysis effort, the best estimate of this correlation was taken to have a correlation coefficient between the log of the pre-remediation load, *lnL*, and the log of the remediation factor, *lnR(t)*, of $p = -0.5$.

For each alternative, remedial action is more effective at higher loadings because each of the remedial actions include a good deal of erosion control, which is more effective at high-flow than low-flow conditions. Extreme flood loadings could damage portions of a remedial action, which could drop its effectiveness. It was assumed that, as part of long-term O&M, any damage to the remedial action would be repaired, as needed, after damaging events.

As a check on sensitivity, Figure D-8 shows the effect of correlation, as measured by p, on the expected value of the post-remediation loadings, *E[F(t)]*. The results are shown for incremental values of p between −1.0 and +1.0 as the ratio of post-remediation loading to the base case loading for $p = -0.5$. The ratios are averaged over the six alternatives. In the early years after remediation, up to 100 years, the effect on post-remediation loading is modest, about 10 percent over the range between no correlation and $p = -0.5$. However, the effect increases with time and becomes moderate to large for long time periods, say, 500 years.

Correlation Between Remedial Effectiveness and Beta, the Rate of Leaching Reduction

Some degree of correlation between remediation factor, expressed in the term *lnRo* and the rate of leaching reduction, beta, presumably exists. This interplay is discussed in Appendix B, Section B.3.3.2, and is thought to be an important effect only for long periods of time (e.g., hundreds of years) after remediation.

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Left (Lower) Side Exceedance Probabilities for a Variety of Probability Distributions as Function of the Number of Standard Deviations from the Mean

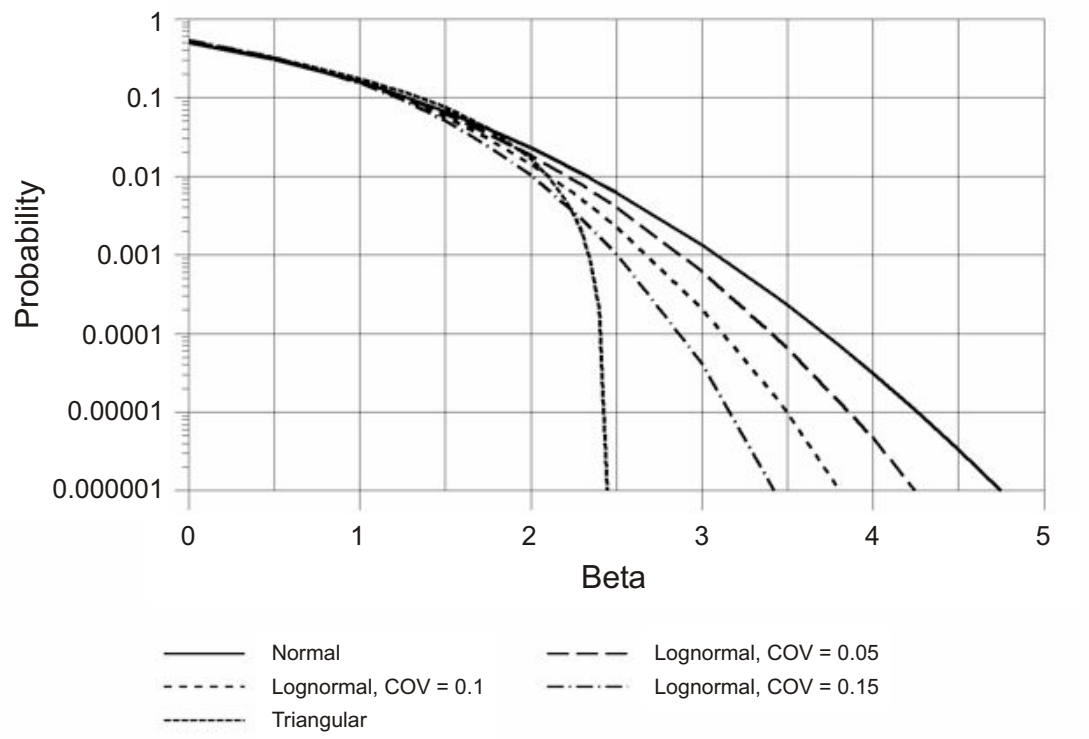

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Figure D-1

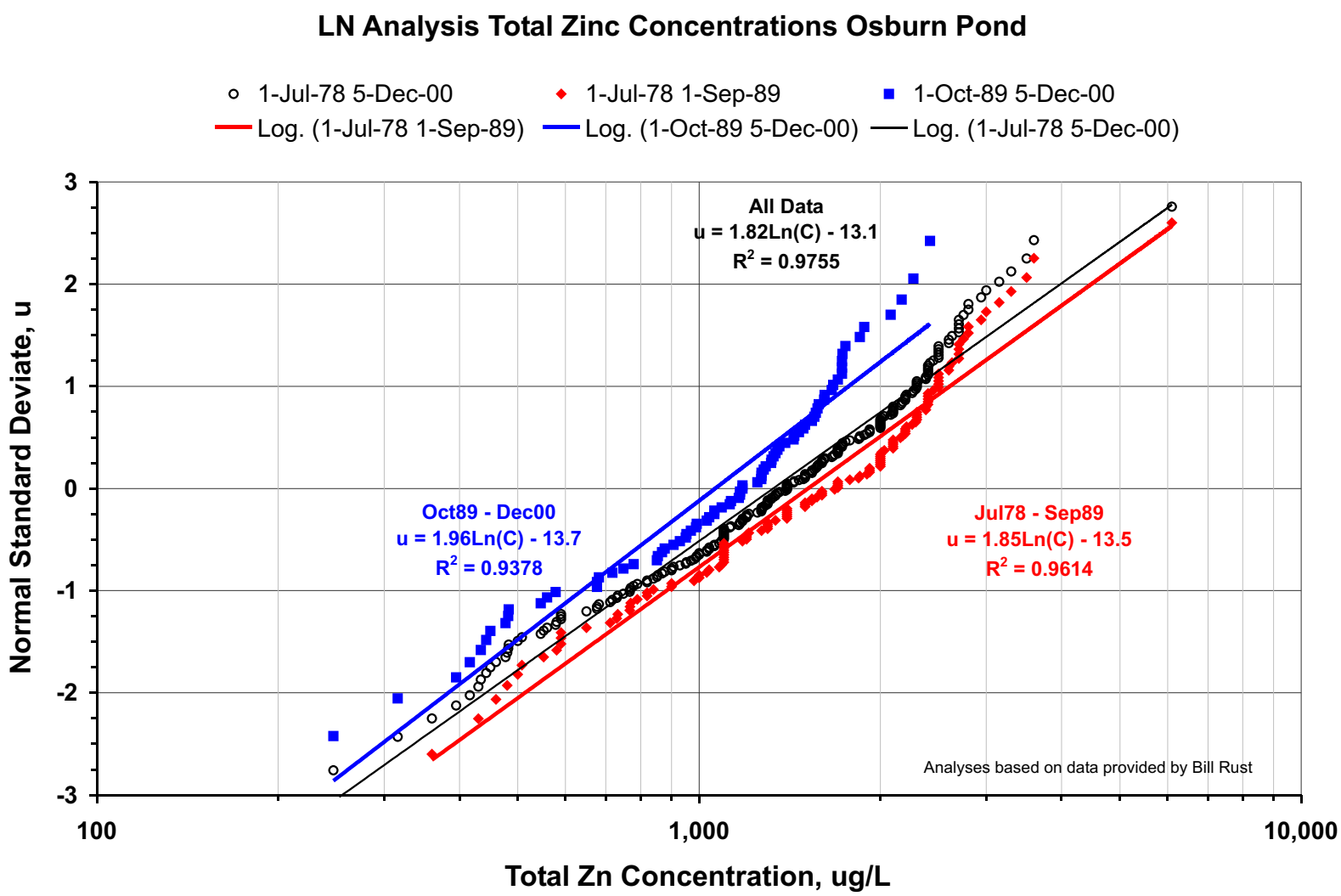

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Figure D-2

95% Non-Exceedance Estimates v. Expected Values of Remediation Factor R Assuming Uncertainty in R Follows Lognormal, Normal, and Beta Distributions

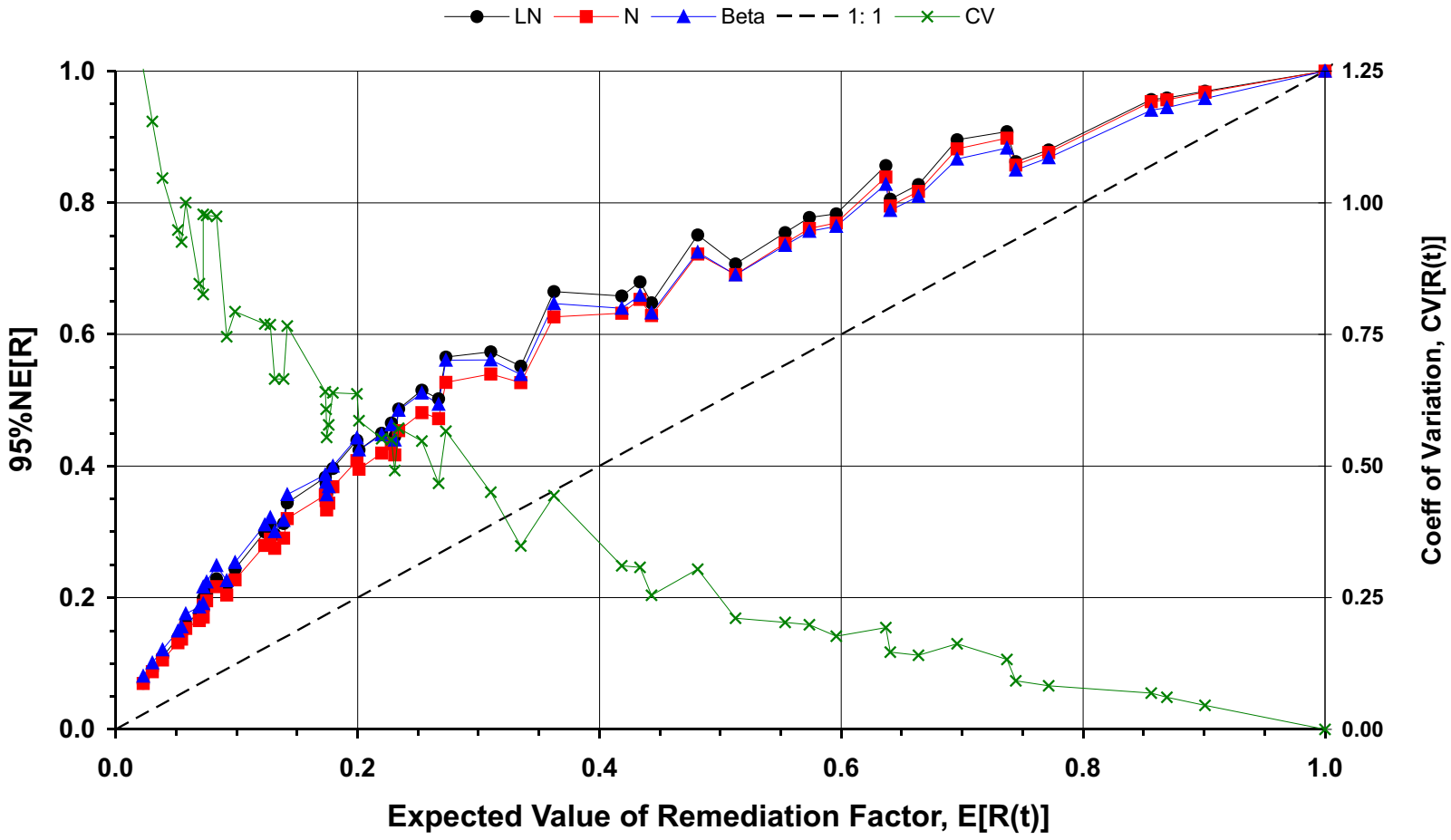

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Figure D-3

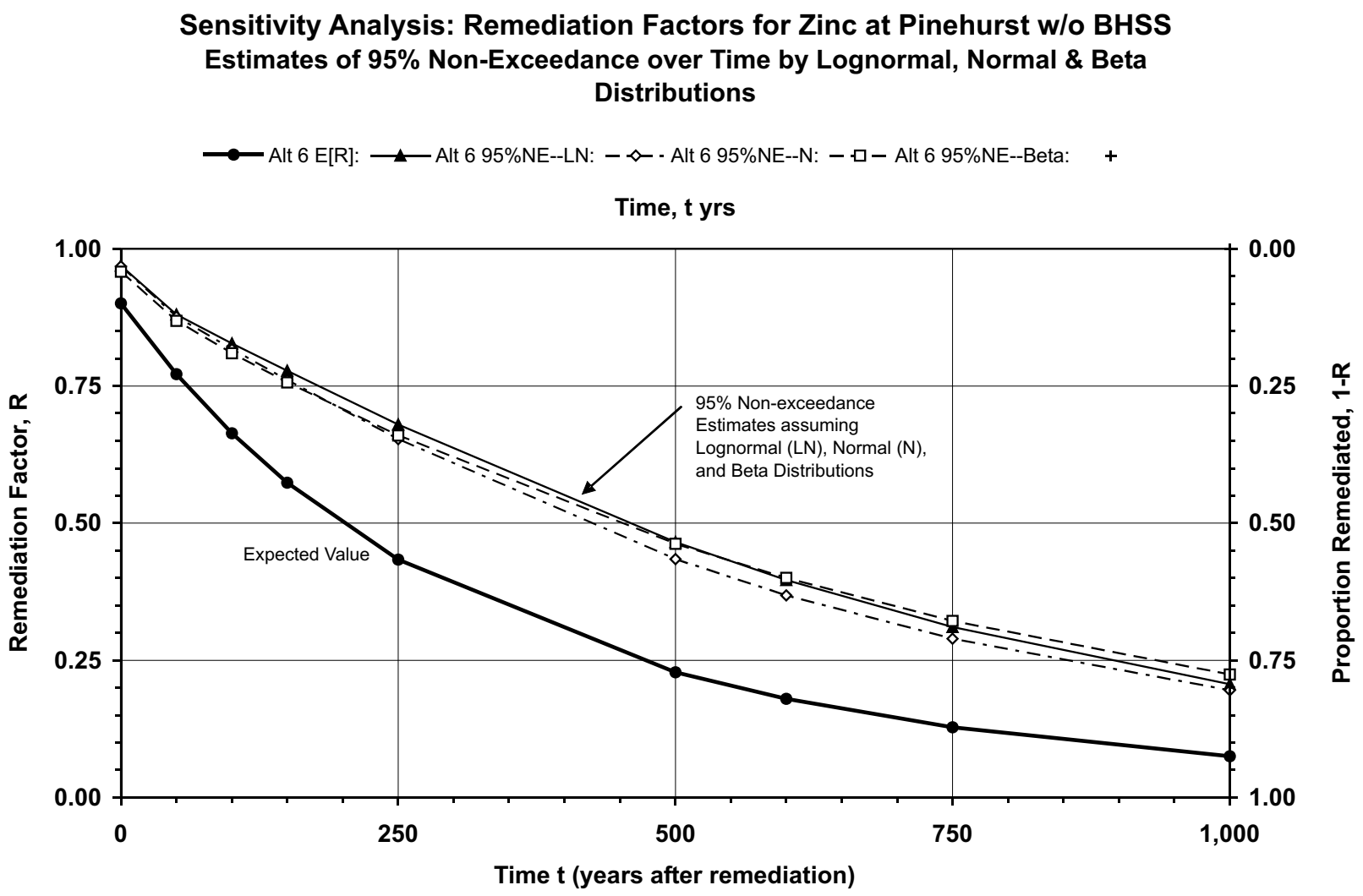

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Figure D-4

Relative Loading Potential (RLP) Sensitivity Analysis Results
Floodplain Sediments

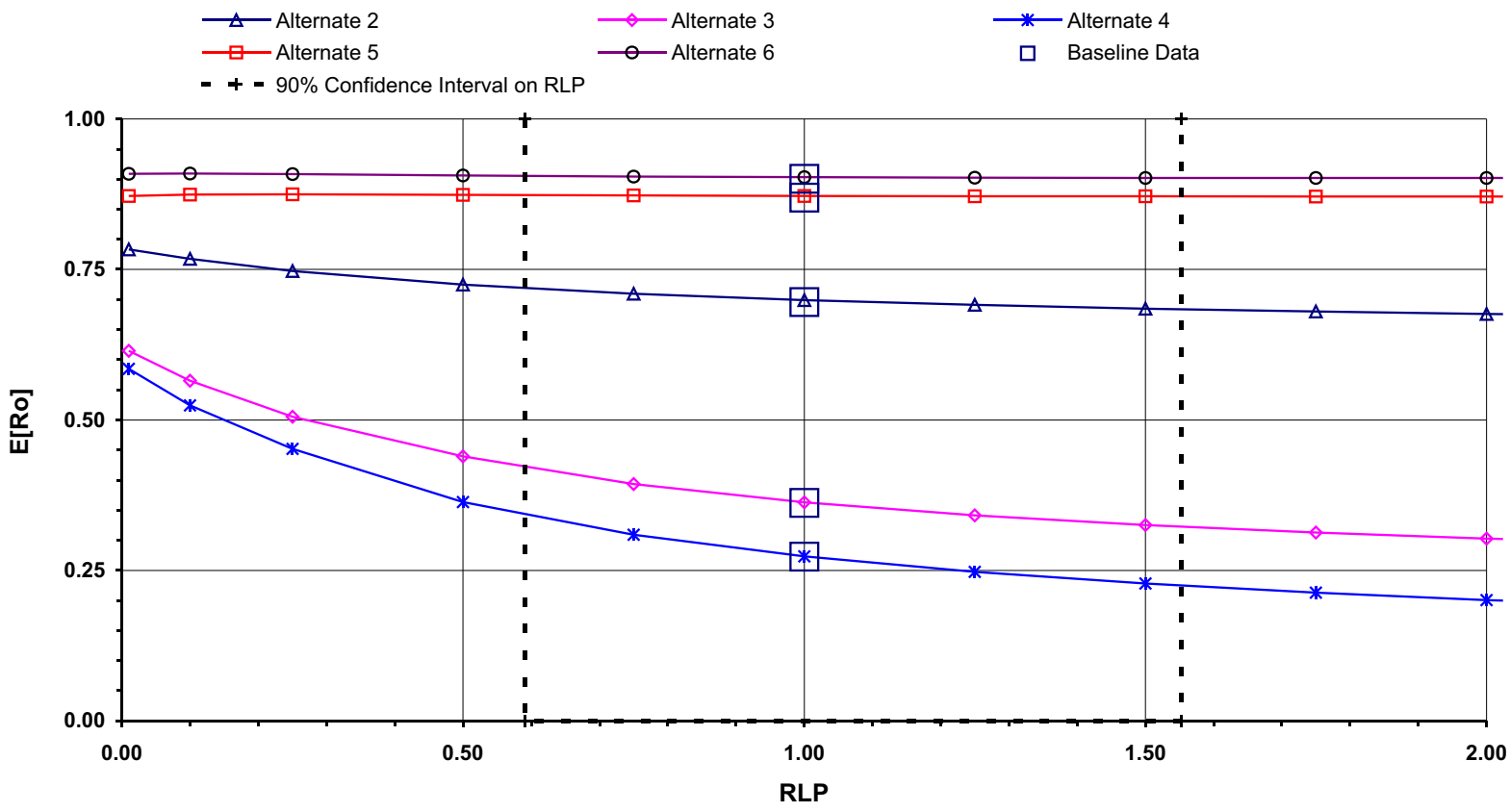

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Figure D-5

(Rij) Sensitivity Analysis Results
Floodplain Sediments: Excavation/Disposal in Repository

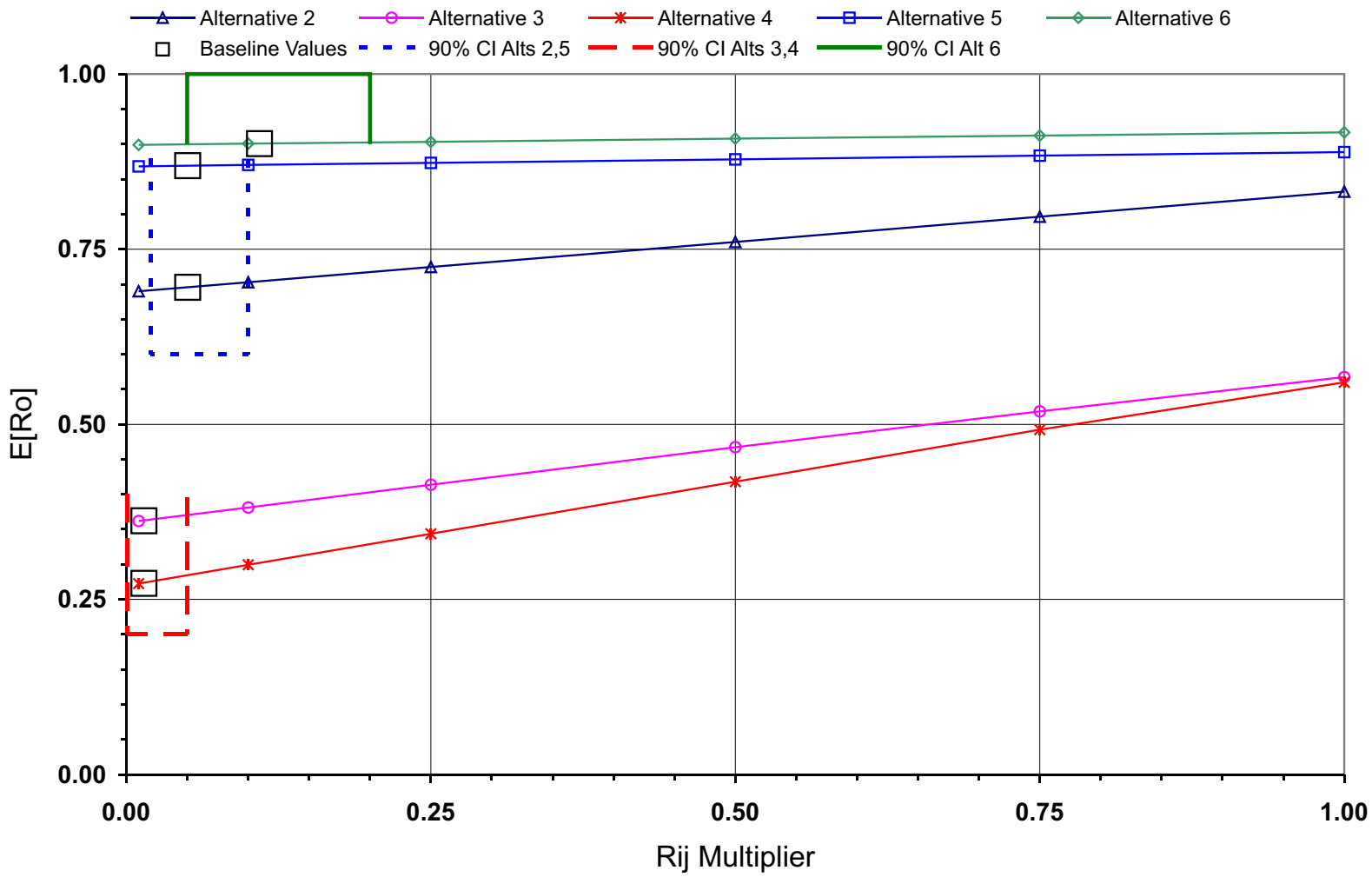

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Figure D-6

Volume (Vij) Sensitivity Analysis Results:
Floodplain Sediments: Excavation/Disposal in Repository

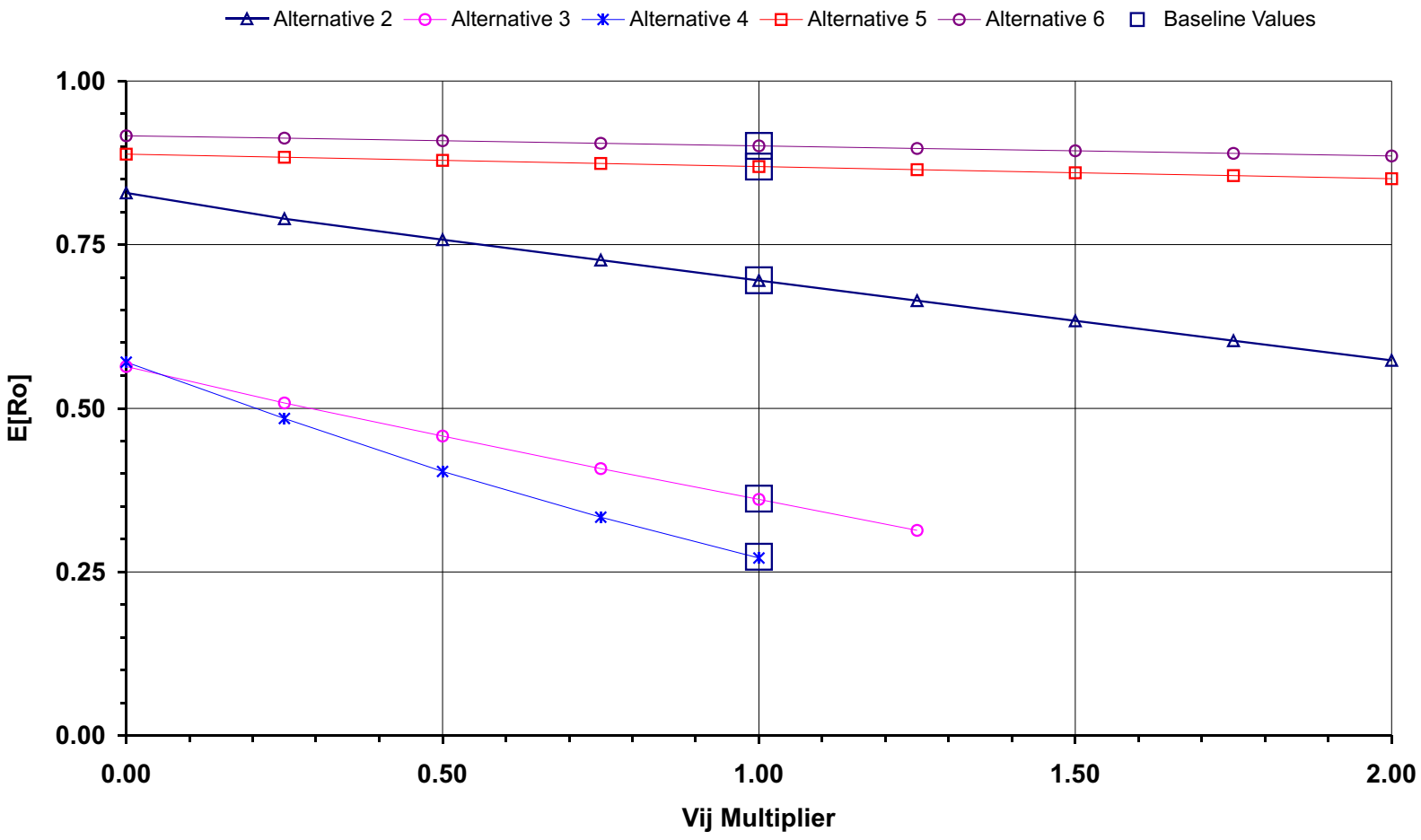

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Figure D-7

Sensitivity Analysis of Post-Remediation Loading F(t) to Correlation Coefficient "p" Between Pre-Remediation Load L and Remediation Factor R(t): Average Ratio of Expected Values E[F(t)] for given p to Base Case E[F(t)] for p = -0.50

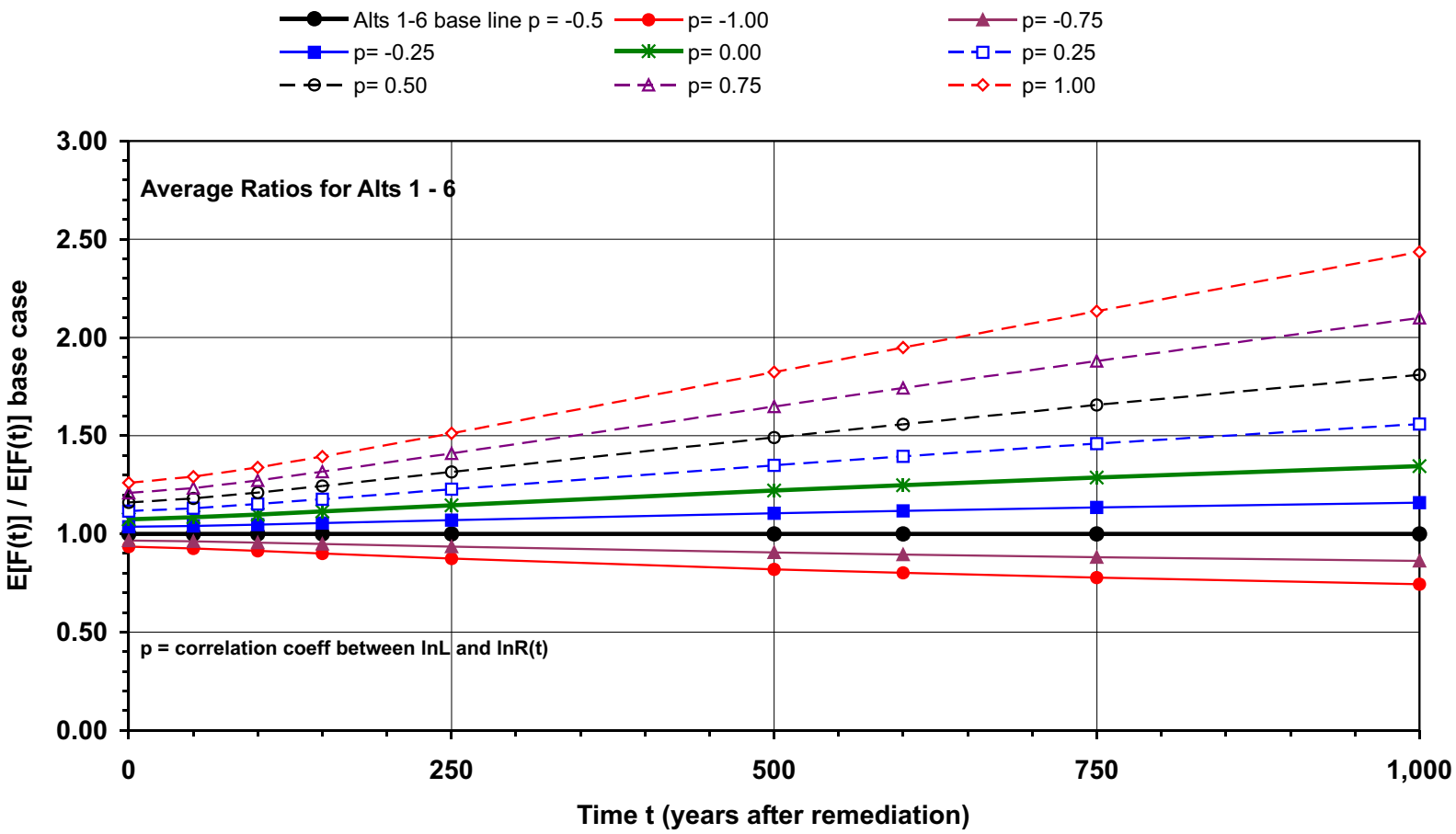

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Figure D-8

Table D-1
Probability of the Excursion of a Random Variable Exceeding Bounds About the Mean Measured in Units, *N*, of the Standard Deviations

N	Chebychev Bound	Normal Lognormal
2	0.25	0.05
3	0.22	0.0026
4	0.06	6.3E-5

Note: The Chebychev bound makes no assumption on the shape of the probability distribution The normal and lognormal case is based on the arithmetic and logarithmic standard deviation, respectively.

Table D-2
Parameter Inventory and Basis for Estimates

Parameter		Basis of Estimation	Rationale
Pre-remediation loadings	*L*	• Historical stream data • Intrinsic variability in time • Epistemic uncertainty in estimates of moments	• As spatial and temporal averages, the source types are statistically smoothed. • Spatial and temporal loading variations within a given source type are described by probability distributions having constant statistical parameters (i.e., expected values and coefficients of variation) over the volumes and time of interest. • Statistical homogeneity for volume (space) followed from segregating individual sources into common source types.
Relative loading potential	*RLP*	• RLPs were judgmentally estimated from consideration of metal concentrations, mobility, and exposure to leaching and erosion and analysis of simple loading models.	• Moments of probability distributions were estimated by considering most probable intervals, and using properties of the lognormal probability density function to calculate first two moments. Statistical homogeneity for time was obtained by making the time of interest one water year. Seasonal and shorter term effects are thus integrated and averaged out.
Remediation factor	R_{ij}	• For each alternative, effectiveness estimates were judgmentally estimated based on an assessment of each remedial action and an engineering interpretation of the range of potential effectiveness. • Estimates for Alternatives 2, 3, and 4 were based on engineering interpretation of the range of potential effectiveness for the typical conceptual designs used in the alternatives, as documented in the FS. These interpretations used qualitative engineering analysis,	• Remediation factor estimates apply to the time immediately following completion of the remedial alternative, but after attenuation of any relatively short-term transient loading effects caused by the remedial construction (e.g., temporarily increased loading due to extensive excavation in stream channels). • The lognormal distributions reflect the aggregate uncertainty in the input estimates of RLPs, source-

Table D-2
Parameter Inventory and Basis for Estimates

Parameter		Basis of Estimation	Rationale
Remediation factor (Cont.)		limited quantitative performance modeling, experience with similar remedial actions, and professional judgment.	type quantities, remediated volumes, remedial action effectiveness for each remedial action, and the source depletion decay rate. Within practical limits, the lognormal distributions quantify the uncertainty in the remediation factors for a given stream monitoring location and remedial alternative.
Source volume	V_j	• Assumed known only approximately	• Values taken from RI.
Remediated volume	V_{ij}	• Assumed known only approximately	• Means and coefficients of variation taken from FS.
Source depletion and natural recovery	ß	• Estimates of natural recovery were based on a model of source depletion that describes the reduction in metal loading and concentration over time as a function of a decay rate.	• For a given monitoring location, the decay rate is estimated as the yearly average pre-remediation metal (zinc) loading divided by the pre-remediation aggregate quantity of metal (zinc) that is available for transport from all upstream source media. Statistical homogeneity for time was obtained by making the time of interest one water year. Seasonal and shorter term effects are thus integrated and averaged out. • Uncertainty was handled by characterizing the pre-remediation loadings and aggregate metal quantity, and thus the decay rate, as lognormal distributions characterized by expected values and coefficients of variation.
Post-remediation loadings		• Calculated parameters, function of *RLPs*, R_{ij}'s, and V_{ij}'s	• Expected values and coefficients of variations of post-remediation loadings are calculated from moments of input variables.

Note: FS - feasibility study (USEPA 2000b)

Table D-3
Summary of Coefficients of Variation for Principal Parameters – Upper Basin*

Parameter	Source Type	Coefficient of Variation
Relative load potential	Impounded tailings	1.76
	Unimpounded tailings	0.80
	Floodplain waste rock	1.59
	Upland waste rock	0.52
	Adits	nil
	Tailings-impacted floodplain sediments	0.30
	Deeper floodplain sediments	0.71
Remediated volumes	Upper basin[a]	0.2 – 0.3
Remediation Factor	Upper basin[a]	0.04 - 1.76
Source Volumes	Impacted floodplain sediments	0.3
	Waste rock	0.3
	Unimpounded tailings	0.30
	Impounded tailings	0.20
	Deeper floodplain sediments	0.52

[a]See Appendix C, Table C-1.
[b]See Appendix C, Table C-2.

* Lower Basin used approach discussed in B.3.3.3 and C.3

Table D-4
Approximate Sensitivities of the Expected Remediation Factor in the PAT1 Model to Relative Loading Potential (RLP_j) for Source Types and Remedial Alternatives, Expressed as the Derivative $dE[R_o]/dRLP_j$

Source Type	Sensitivity: $dE[R_o]/dRLP_j$				
	Alt 2	Alt 3	Alt 4	Alt 5	Alt 6
Floodplain sediments	-0.04	-0.11	-0.14	0.00	0.00
Deeper floodplain sediments: not removed	0.22	0.50	0.58	0.07	0.06
Tailings: impounded in inactive facilities	-0.17	-0.12	-0.10	-0.20	-0.12
Tailings: impounded in active facilities	0.12	-0.02	-0.03	0.04	0.03
Tailings: unimpounded	-0.02	-0.01	-0.01	-0.01	-0.03
Waste rock with loading potential	-0.06	-0.11	-0.07	-0.03	-0.07
Waste rock without loading potential	0.12	0.22	0.11	0.04	0.03

Notes:
Alt - Alternative
PAT1 - Probabilistic Analysis Tool 1

Table D-5
Approximate Sensitivities of the Expected Remediation Factor in the PAT1 Model to the Remedial Effectiveness (R_{jj}) for Source Types and Remedial Alternatives, Expressed as the Derivative $dE[R_o]/dR_{ij}$

Source Type	Sensitivity: $dE[R_o]/dR_{ij}$				
	Alt 2	Alt 3	Alt 4	Alt 5	Alt 6
Floodplain sediments: excavation and disposal in repository	0.14	0.21	0.30	0.02	0.02
Floodplain sediments: hydraulic isolation at discrete facilities	0.03	0.17	0.14	-	-
Floodplain sediments: hydraulic isolation of stream reaches	-	0.11	0.14	-	-
Impounded tailings at inactive facilities: capping and hydraulic isolation	0.03	0.03	-	-	-
Impounded tailings at active facilities: hydraulic isolation	-	0.05	0.06	-	-
Impounded tailings at inactive facilities: excavation and disposal in repository	-	0.001	0.05	0.002	-
Unimpounded tailings: excavation and disposal in repository	0.002	0.03	0.04	0.004	0.002
Unimpounded tailings: cover and revegetate	-	-	-	0.003	0.03
Waste rock with loading potential: cap	0.002	0.02	0.0001	0.0002	-
Waste rock with loading potential: excavation and disposal in repository	-	0.01	0.03	0.0001	-
Waste rock with loading potential: regrade and cover	0.02	0.003	0.001	0.01	0.005
Upland waste rock: regrade and cover	-	0.0002	0.001	IE-06	4E-06
Waste rock with loading potential: regrade/cover	0.02	0.003	0.001	0.01	0.005
Adit discharge: active treatment	-	0.06	0.07	-	-
Adit discharge: passive treatment	0.06	0.01	0.01	0.07	0.05

Notes:
Alt - Alternative
PAT1 - Probabilistic Analysis Tool 1

Table D-6
Approximate Sensitivities of the Expected Remediation Factor in the PAT1 Model to the Volume of Remediated Source Material (V_{ij}) for Source Types and Remedial Alternatives, Expressed as the Derivative $dE[R_o]/dV_{ij}$

Source Type	Sensitivity: $dE[R_o]/dV_{ij}$				
	Alt 2	Alt 3	Alt 4	Alt 5	Alt 6
Floodplain sediments: excavation and disposal in repository	-0.12	-0.19	-0.30	-0.02	-0.02
Floodplain sediments: hydraulic isolation of stream reaches	-	-0.08	-0.10	-	-
Deeper Floodplain sediments: no remedation	0.04	0.09	0.11	0.01	0.01
Impounded tailings at inactive facilities: capping and hydraulic isolation	-0.03	-0.03	-	-	-
Unimpounded tailings: excavation and disposal in repository	-0.002	-0.03	-0.03	-0.004	-0.002
Unimpounded tailings: cover and revegetate	-	-	-	-0.002	-0.01
Waste rock: excavation and disposal in repository	-	-0.01	-0.03	-0.0001	-
Waste rock with loading potential: cap	-0.002	-0.02	-0.0001	0.0002	-
Waste rock with loading potential: regrade and cover	-0.01	-0.002	-0.001	-0.004	-0.003
Upland waste rock: regrade and cover	-	-0.0001	-0.001	-7E-07	-2E-06
Adit discharge: passive	-0.06	-0.01	-0.01	-0.06	-0.04

Notes:
Alt - Alternative
PAT1 - Probabilistic Analysis Tool 1
V_{ij} - expressed as fraction of baseline V_{ij}

Table D-7
Effect of Correlation Between Pre- and Post-remediation Relative Remediated Volume on Post-remediation Loading

	Remedial Alternative				
	2	3	4	5	6
base case p:	0.900	0.376	0.364	0.988	0.993
E[R]:	0.696	0.362	0.273	0.869	0.901
CV[R]:	0.163	0.444	0.566	0.061	0.045

Alternative 2			Alternative 3			Alternative 4			Alternative 5			Alternative 6		
p	E[R]	CV[R]	p	E[R]	CV[R]	p	E[R]	CV[R]	p	E[R]	CV[R]	p	E[R]	CV[R]
0.00	0.761	0.479	0.00	0.378	0.552	0.00	0.288	0.688	0.00	0.965	0.488	0.00	0.999	0.481
0.25	0.742	0.412	0.25	0.367	0.482	0.25	0.278	0.606	0.25	0.940	0.417	0.25	0.973	0.411
0.50	0.724	0.335	0.376	0.362	0.444	0.364	0.273	0.566	0.50	0.915	0.337	0.50	0.948	0.332
0.75	0.706	0.241	0.50	0.357	0.404	0.50	0.267	0.517	0.75	0.891	0.236	0.75	0.924	0.232
0.900	0.696	0.163	0.75	0.347	0.314	0.75	0.258	0.419	0.988	0.869	0.061	0.993	0.901	0.045
1.00	0.689	0.079	1.00	0.337	0.193	1.00	0.248	0.300	1.00	0.868	0.034	1.00	0.900	0.023

Note: Correlation coefficient "p" between lnRLV' and lnRLV" used in estimating R_o = RLV"/RLV' (see Appendix B, Section B.3.3.1)